Moshpit SGD: Communication-Efficient Decentralized Training on Heterogeneous Unreliable Devices

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Moshpit SGD: Communication-Efficient Decentralized Training on Heterogeneous Unreliable Devices

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The talk is based on our NeurIPS 2021 paper
Outline

1. Motivation
2. Moshpit All-Reduce
3. Moshpit SGD
1. Motivation
MNIST

Fully connected NN with 3 hidden layers
MNIST

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Requires several minutes to train if executed on a good enough laptop
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Common Crawl

Books Corpus

Wikipedia
MNIST

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GPT-3
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Requires several years to train if executed on top-of-the-line GPU server
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It is mandatory to have efficient distributed algorithms
Faulty Workers
Instead of one „powerful“ machine, multiple machines are used
Faulty Workers

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- Some machines can fault to execute the communication protocol at arbitrary stages of the work.
Faulty Workers

Instead of one "powerful" machine, multiple machines are used.

Some machines can fault to execute the communication protocol at arbitrary stages of the work.

It is important to have fault-tolerant distributed methods.
With Parameter-Server (PS):

✔ Simple and widely applicable approach

✘ Not scalable: for large number of participants the communication is a bottleneck

Devices send and receive full vectors
Communication

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Without PS via All-Reduce:

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With Parameter-Server (PS):

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- Not scalable: for large number of participants the communication is a bottleneck

Without PS via All-Reduce:

- Scalable approach
- Not robust to faults
Communication

- **With Parameter-Server (PS):**
  - ✔️ Simple and widely applicable approach
  - ❌ Not scalable: for large number of participants the communication is a bottleneck

- **Without PS via All-Reduce:**
  - ✔️ Scalable approach
  - ❌ Not robust to faults

- **Without PS via gossip:**
Communication

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  - ✔️ Simple and widely applicable approach
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- **Without PS via All-Reduce:**
  - ✔️ Scalable approach
  - ✗ Not robust to faults

- **Without PS via gossip:**

\[
g_i^{k+1} = \sum_{j=1}^{n} M_{i,j} g_j^k
\]
With Parameter-Server (PS):

- **✓ Simple and widely applicable approach**
- **✗ Not scalable: for large number of participants the communication is a bottleneck**

Without PS via All-Reduce:

- **✓ Scalable approach**
- **✗ Not robust to faults**

Without PS via gossip:

\[ g_{i}^{k+1} = \sum_{j=1}^{n} M_{ij} g_{j}^{k} \]

Mixing matrix defines the communication pattern
Communication

With Parameter-Server (PS):
- ✔ Simple and widely applicable approach
- ✗ Not scalable: for large number of participants the communication is a bottleneck

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- ✔ Scalable approach
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Without PS via gossip:
- Devices send and receive full vectors
- Mixing matrix defines the communication pattern
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- ✔ Simple and widely applicable approach
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Without PS via All-Reduce:

- ✔ Scalable approach
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Without PS via gossip:

- ✔ Scalable approach
- ✗ Inevitable dependence on mixing matrix and graph structure

Devices send and receive full vectors

Mixing matrix defines the communication pattern
2. Moshpit All-Reduce
All-Reduce protocols are fragile: the fault of 1 worker affects all other workers
Moshpit All-Reduce: Main Idea

- All-Reduce protocols are fragile: **the fault of 1 worker affects all other workers**
- The idea: execute **All-Reduce in small groups**
Moshpit All-Reduce: Main Idea

- All-Reduce protocols are fragile: the fault of 1 worker affects all other workers

- The idea: execute All-Reduce in small groups

  The fault of one peer affects only its group
Moshpit All-Reduce: Ideal Case

Workers form $d$ dimensional hypercube with $M$ workers along each axis

$d = 2, M = 3$
Algorithm 1 Moshpit All-Reduce (for $i$-th peer)

**Input:** parameters $\{\theta_j\}_{j=1}^N$, number of peers $N$, $d$, $M$, number of iterations $T$, peer index $i$

$\theta^0_i := \theta_i$

$C^0_i := \text{get_initial_index}(i)$

for $t \in 1 \ldots T$ do

$\text{DHT}[C^{t-1}_i,t].\text{add}(\text{address}_i)$

$\text{Matchmaking()}$ // wait for peers to assemble

$peers_t := \text{DHT}.\text{get}([C^{t-1}_i,t])$

$\theta_t^i, c_t^i := \text{AllReduce}(\theta^{t-1}_i, peers_t)$

$C^t_i := (C^{t-1}_i[1:], c_t^i)$ // same as eq. (1)

end for

Return $\theta^T_i$

get_initial_index $(i) = ([i/M^{d-1}] \mod M)_{j \in \{1, \ldots, d\}}$

$$C^t_i := (c_t^{i-d+1}, c_t^{i-d+2}, \ldots, c_t^i)$$

Distributed Hash Table — an efficient decentralized data structure
If $N = M^d$ and there are no faults, then Moshpit All-Reduce finds an exact average after $d$ steps.
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**Correctness:** if all workers have a non-zero probability of successfully running a communication round and the order of peers is random, then all local vectors converge to the global average with probability 1:
Moshpit All-Reduce: Theoretical Properties

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\forall i \quad \left\| \theta_i^t - \frac{1}{N} \sum_i \theta_i^0 \right\|_2^2 \xrightarrow{t \to \infty} 0
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- **Exponential convergence to the average:** for a version of Moshpit All-Reduce with random splitting into $r$ groups at each step, we have...
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- **Exponential convergence to the average:** for a version of Moshpit All-Reduce with random splitting into \( r \) groups at each step, we have

\[
\mathbb{E} \left[ \frac{1}{N} \sum_{i=1}^{N} \left\| \theta_i^T - \bar{\theta} \right\|^2 \right] = \left( \frac{r-1}{N} + \frac{r}{N^2} \right)^T \frac{1}{N} \sum_{i=1}^{N} \left\| \theta_i - \bar{\theta} \right\|^2
\]
Moshpit All-Reduce: Experiments

- We verify the performance gains in a controlled setting.
- With non-zero failure probability, All-Reduce takes too many retries!
- On the other hand, Gossip-based methods converge very slowly.
- Moshpit All-Reduce outperforms baselines with $p > 0$ and gets the average in two rounds with $p = 0$. 

![Graphs showing performance comparison between Moshpit All-Reduce and baselines under different conditions.](image-url)
3. Moshpit SGD
The Problem

\[
\min_{x \in \mathbb{R}^n} f(x)
\]

- Function \( f(x) \) is available through stochastic gradients only
- Each worker has an access to the stochastic gradients of \( f(x) \)
Moshpit SGD

\[ x_i^{k+1} = \begin{cases} 
  x_i^k - \gamma g_i^k, & \text{if } k + 1 \mod \tau \neq 0 \\
  \text{Moshpit All-Reduce}_{j \in P_{k+1}} (x_j - \gamma g_j^k), & \text{if } k + 1 \mod \tau = 0 
\end{cases} \]

Number of active workers at iteration \( k+1 \)
Moshpit SGD

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\end{cases} \]

Local-SGD with Moshpit All-Reduce instead of averaging

Number of active workers at iteration \( k+1 \)
Assumptions

Homogeneity:

\[ f_1(x) = f_2(x) = \ldots = f_N(x) = f(x) \]
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- Bounded variance:

  \[ \mathbb{E} \left[ \left\| g_i^k - \nabla f_i(x_i^k) \right\|^2 \mid x_i^k \right] \leq \sigma^2 \]
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  \[ \mathbb{E} \left[ \left\| g_i^k - \nabla f_i \left( x_i^k \right) \right\|^2 | x_i^k \right] \leq \sigma^2 \]

- **Effect of peers’ vanishing is bounded:**
  \[ \mathbb{E} \left[ \langle x^{k+1} - \hat{x}^{k+1}, x^{k+1} + \hat{x}^{k+1} - 2x^* \rangle \right] \leq \Delta^{k}_{pv} \]
Assumptions

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\[ x^{k+1} = \frac{1}{N_{k+1}} \sum_{i \in P_{k+1}} x_i^{k+1} \]
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\[
\begin{align*}
N_k &= |P_k| \\
x^{k+1} &= \frac{1}{N_{k+1}} \sum_{i \in P_{k+1}} x_i^{k+1}
\end{align*}
\]
Assumptions

Homogeneity:

$$f_1(x) = f_2(x) = \ldots = f_N(x) = f(x)$$

Bounded variance:

$$\mathbb{E} \left[ \left\| g_i^k - \nabla f_i(x_i^k) \right\|^2 \mid x_i^k \right] \leq \sigma^2$$

Effect of peers’ vanishing is bounded:

$$\mathbb{E} \left[ \langle x^{k+1} \rangle - \hat{x}^{k+1} , x^{k+1} \rangle + \hat{x}^{k+1} - 2x^* \right] \leq \Delta^k_{pv}$$

$$N_k = |P_k|$$

$$x^{k+1} = \frac{1}{N_{k+1}^{\text{PV}}} \sum_{i \in P_{k+1}} x_i^{k+1}$$

$$\hat{x}^{k+1} = \frac{1}{N_k} \sum_{i \in P_k} \left( x_i^k - \gamma g_i^k \right)$$
Function $f$ is $\mu$-strongly convex
Assumptions

- Function $f$ is $\mu$-strongly convex

- Averaging quality:

$$\mathbb{E} \left[ \frac{1}{n_{a\tau}} \sum_{i \in P_{a\tau}} \| x_i^{a\tau} - x^{a\tau} \|^2 \right] \leq \gamma^2 \delta_{aq}^2$$
Moshpit SGD finds $\hat{x}$ such that $\mathbb{E} [f(\hat{x}) - f(x^*)] \leq \varepsilon$ after
Moshpit SGD: Complexity

Moshpit SGD finds $\hat{x}$ such that $\mathbb{E} \left[ f(\hat{x}) - f(x^*) \right] \leq \varepsilon$ after

$$\tilde{O} \left( \frac{L}{(1 - \delta_{pv,1}) \mu} + \frac{\delta_{pv,2}^2 + \sigma^2/n_{\min}}{(1 - \delta_{pv,1}) \mu \varepsilon} + \sqrt{\frac{L \left( (\tau - 1) \sigma^2 + \delta_{aq}^2 \right)}{(1 - \delta_{pv,1})^2 \mu^2 \varepsilon}} \right)$$ iterations when $\mu > 0$
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$$O \left( \frac{LR_0^2}{\varepsilon} + \frac{R_0^2 (\delta_{pv,2}^2 + \sigma^2 / n_{\text{min}})}{\varepsilon^2} + \frac{R_0^2 \sqrt{L ((\tau - 1) \sigma^2 + \delta_{aq}^2)}}{\varepsilon^{3/2}} \right)$$ iterations when $\mu = 0$
Moshpit SGD: Complexity

Moshpit SGD finds $\hat{x}$ such that $\mathbb{E}[f(\hat{x}) - f(x^*)] \leq \varepsilon$ after

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If $\delta_{pv,1} \leq 1/2$, $N_{\text{min}} = \Omega(N)$, $\delta_{pv,2}^2 = O(\sigma^2/N_{\text{min}})$, $\delta_{aq}^2 = O((\tau - 1)\sigma^2)$, then

the complexity of Moshpit SGD matches the complexity of centralized Local-SGD
Moshpit SGD: ResNet-50 on Imagenet

- We evaluate and several baselines in two environments
- (16 nodes with 1xV100 and 64 workers with 81 different GPUs)
- Comparable to All-Reduce in terms of iterations, faster in terms of time
- Decentralized methods run faster, but achieve worse results
Moshpit SGD: ALBERT on BookCorpus

- Baseline: All-Reduce on 8 V100
- Moshpit SGD: 66 preemptible GPUs
- Cost of spot instances are much smaller, yet we converge 1.5x faster
4. Conclusion
Summary

- We propose a simple method for communication-efficient distributed training
- Built-in fault tolerance, convergence similar to standard methods
Some of My Recent Works

EG, Marina Danilova, Innokentiy Shibaev, Pavel Dvurechensky, Alexander Gasnikov
Near-Optimal High Probability Complexity Bounds for Non-Smooth Stochastic Optimization with Heavy-Tailed Noise
arXiv:2106.05958

EG*, Alexander Borzunov*, Michael Diskin, Max Ryabinin
Secure Distributed Training at Scale
arXiv:2106.11257

Ilyas Fatkhullin, Igor Sokolov, EG, Zhisu Li, Peter Richtárik
EF21 with Bells & Whistles: Practical Algorithmic Extensions of Modern Error Feedback
arXiv:2110.03294

EG, Nicolas Loizou, Gauthier Gidel
Extragradient Method: O(1/K) Last-Iterate Convergence for Monotone Variational Inequalities and Connections With Cocoercivity
arXiv:2110.04261

EG, Hugo Berard, Gauthier Gidel, Nicolas Loizou
Stochastic Extragradient: General Analysis and Improved Rates
arXiv:2111.08611