Byzantine Robustness and Partial Participation Can Be Achieved at Once: Just Clip Gradient Differences

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G. Malinovsky, P. Richtárik, S. Horváth, E. Gorbunov. *Byzantine Robustness and Partial Participation Can Be Achieved at Once: Just Clip Gradient Differences* (arXiv:2311.14127)

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Outline

1. Byzantine-Robust Training

2. Robust Aggregation

3. Ingredient 1: Variance Reduction

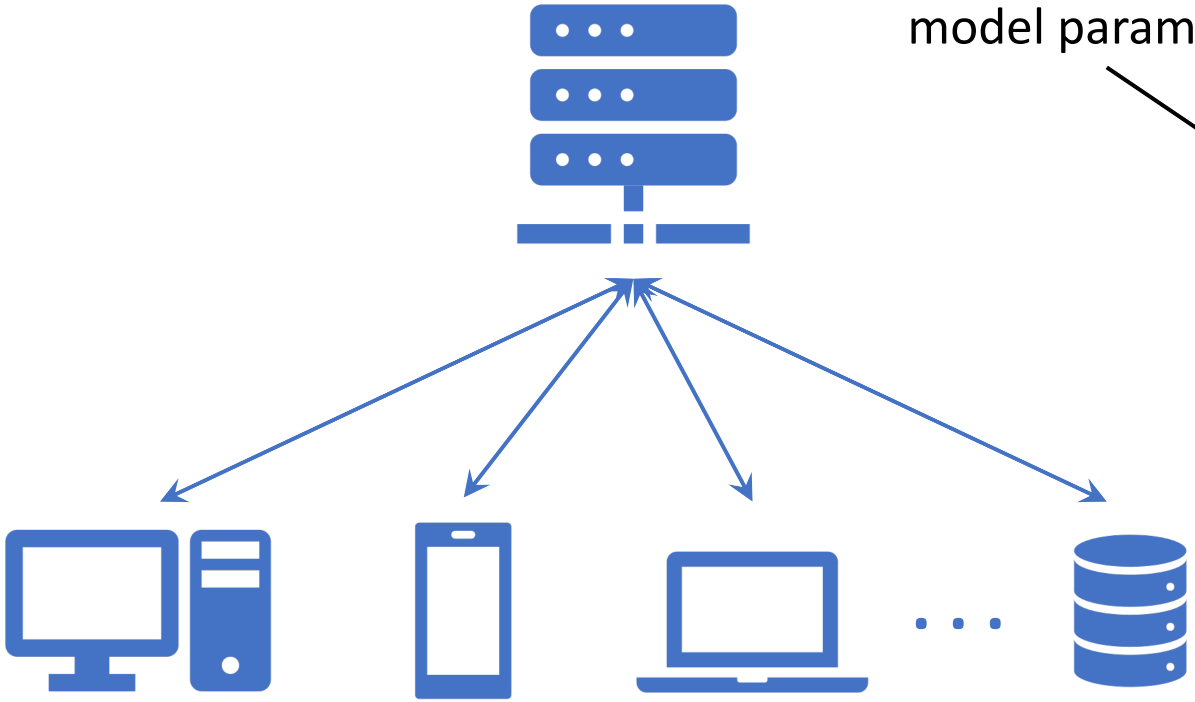
4. Partial Participation of Clients

5. Ingredient 2: Clipping

6. New Method

Byzantine-Robust Training

The Problem

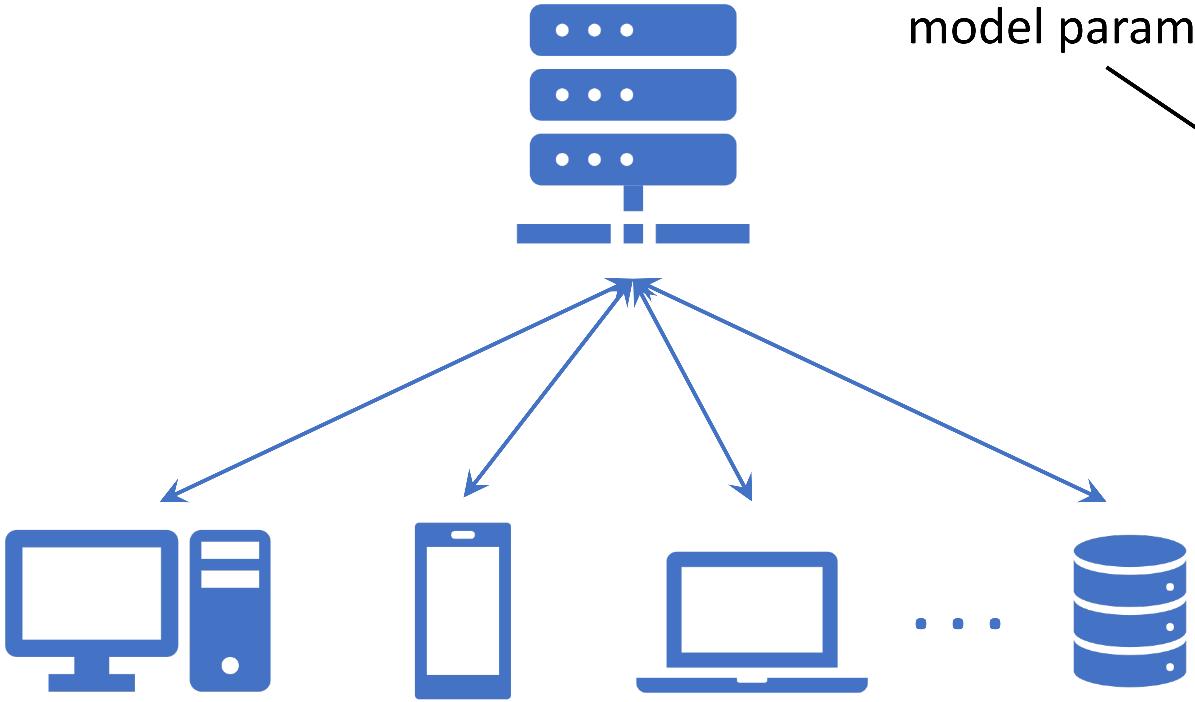


model parameters

$$\min_{x \in \mathbb{R}^d} \left\{ f(x) := \frac{1}{n} \sum_{i=1}^{n} f_i(x) \right\}$$

# of parameters

The Problem



n workers/clients

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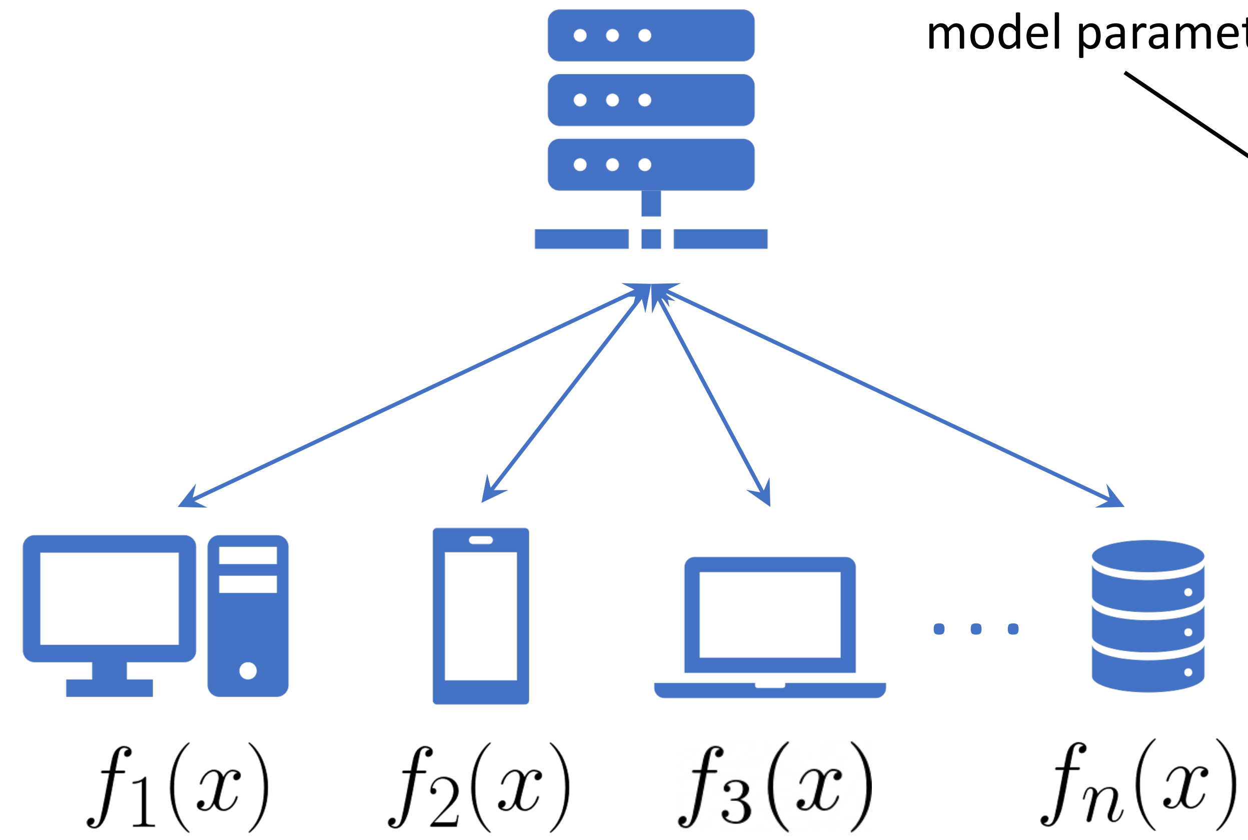

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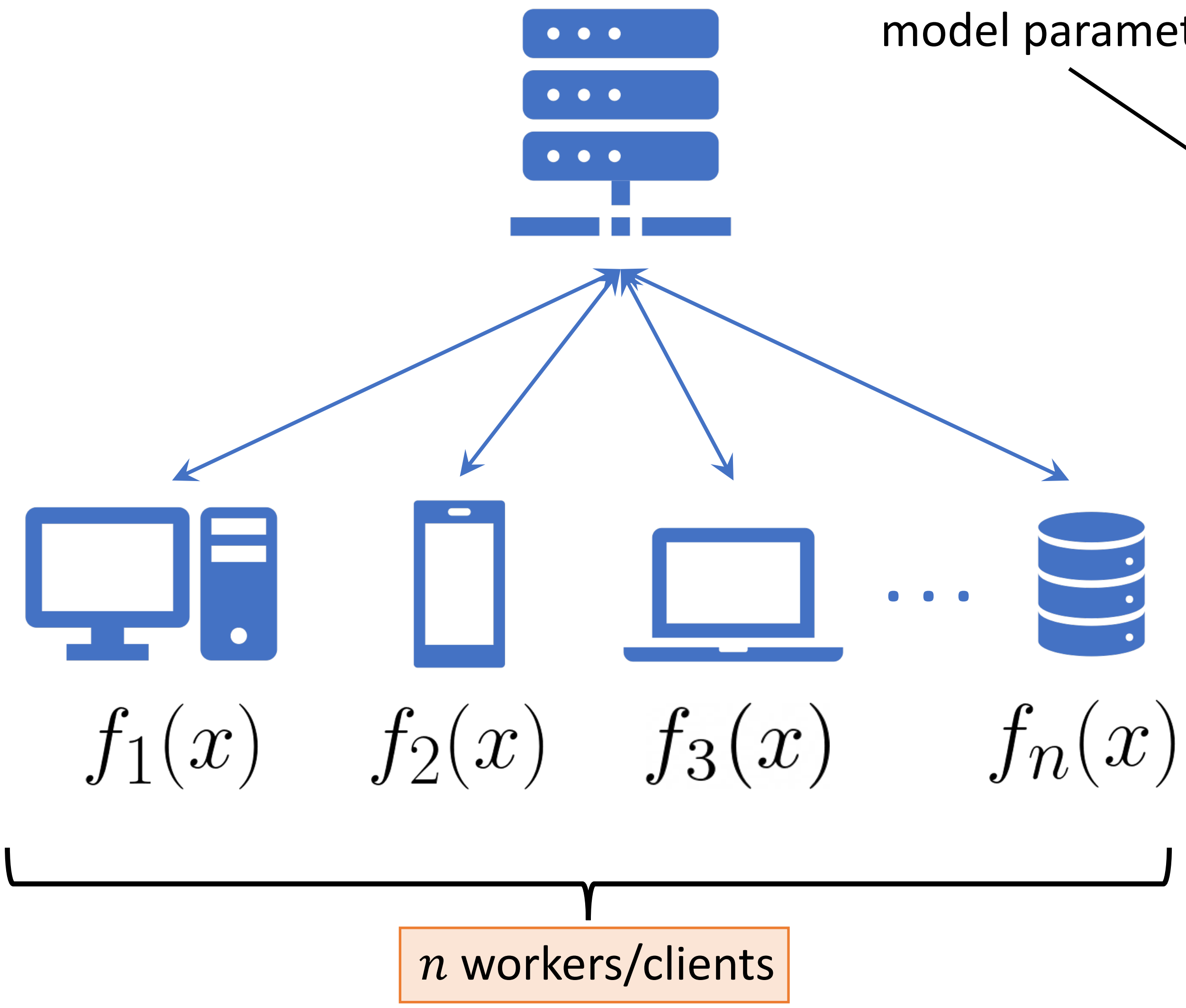

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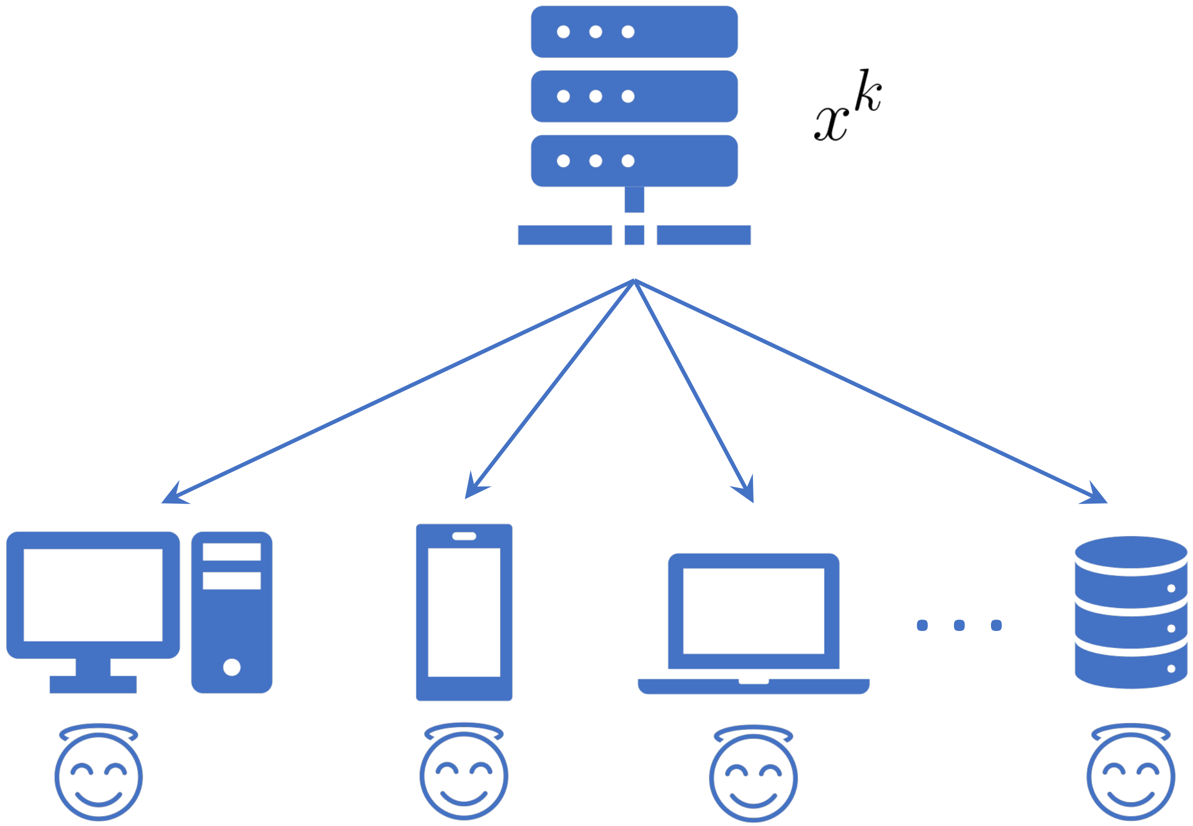
Key features:

- The problem is hard to solve for one client
- Clients do not know each other

Parallel SGD

Iteration k:

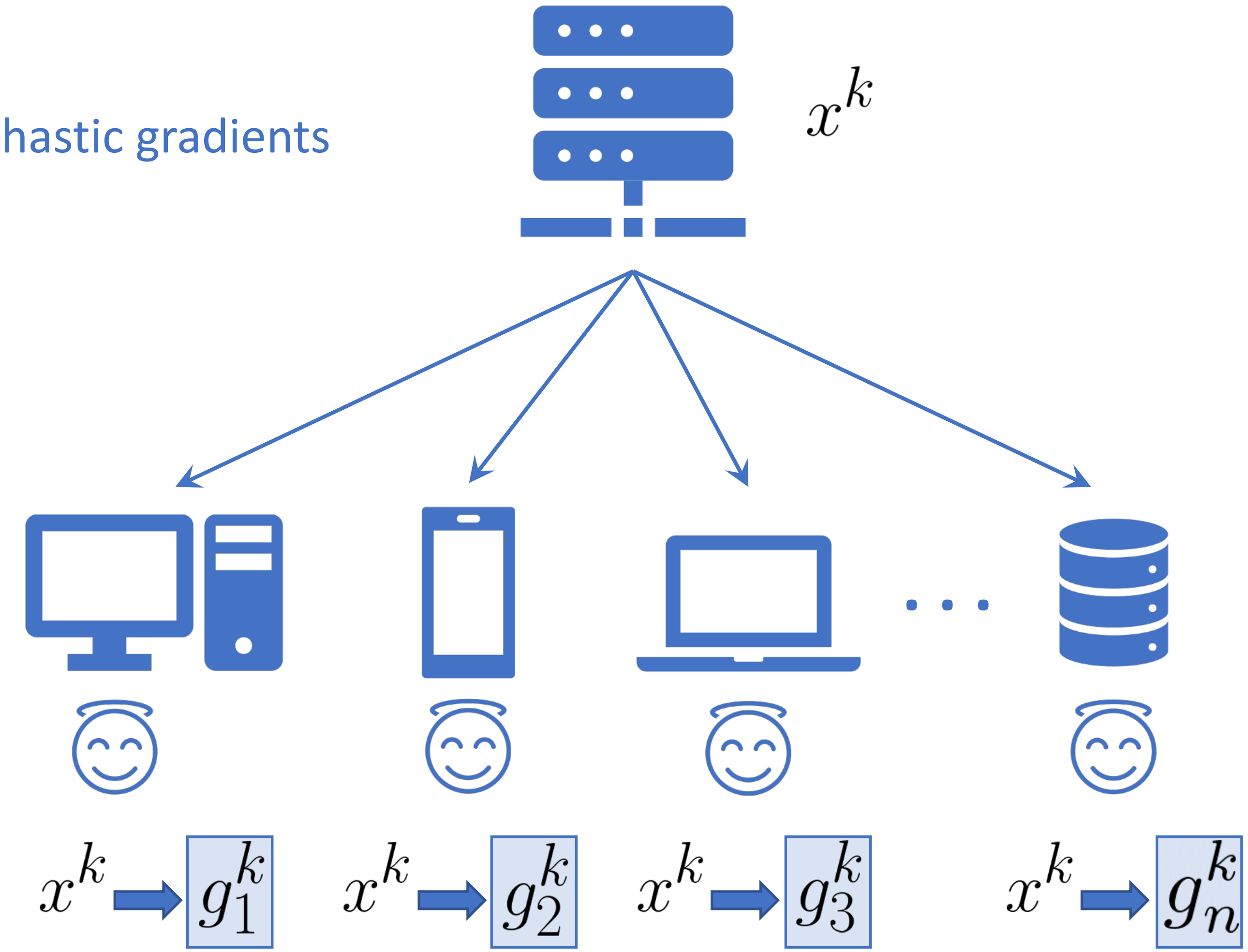
1. Server broadcasts x^k



Parallel SGD

Iteration k:

1. Server broadcasts x^k
2. Workers compute stochastic gradients



$$\mathbb{E}_k[\boxed{g_i^k}] = \nabla f_i(x^k)$$

Parallel SGD

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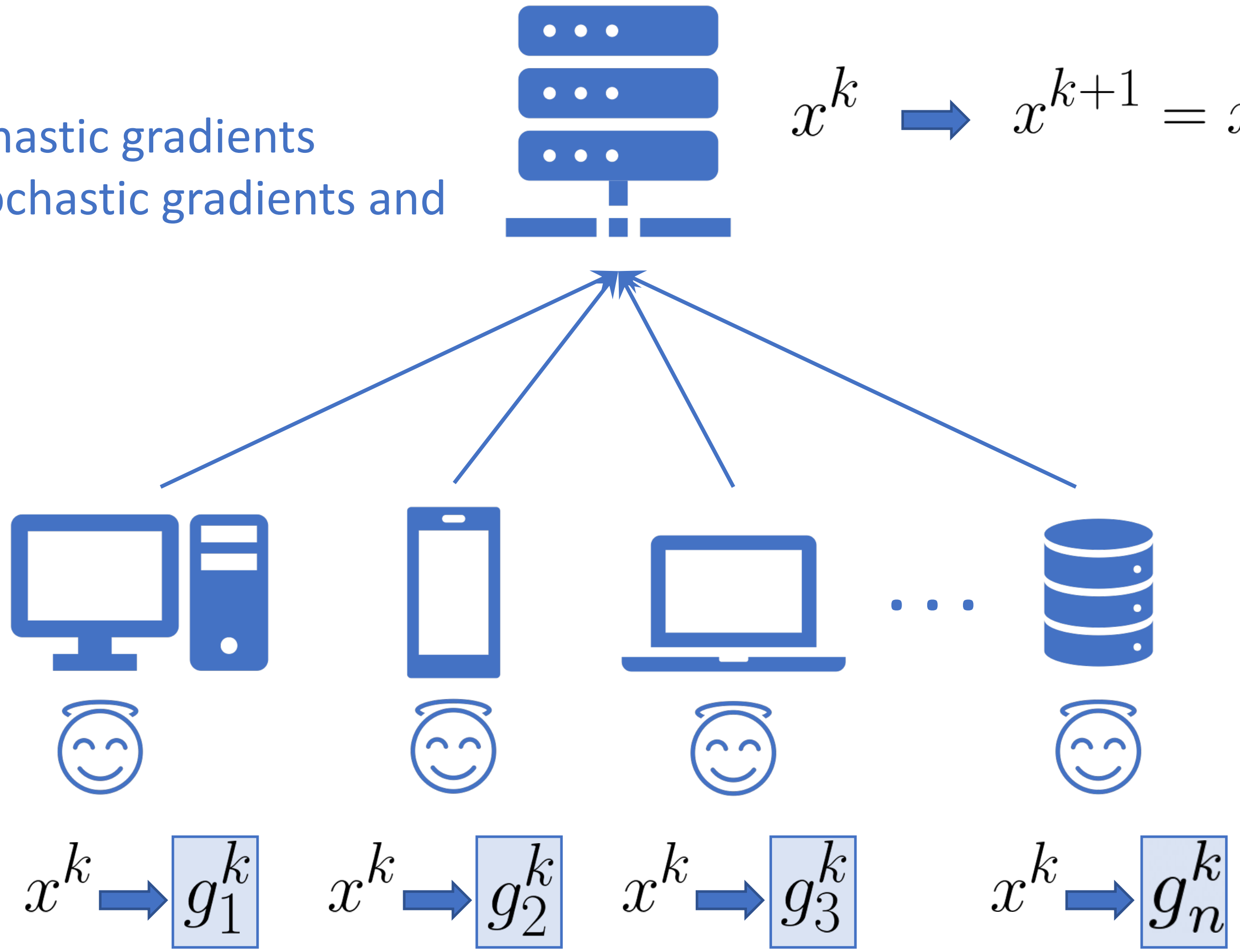
3. Server averages the stochastic gradients and makes an SGD step



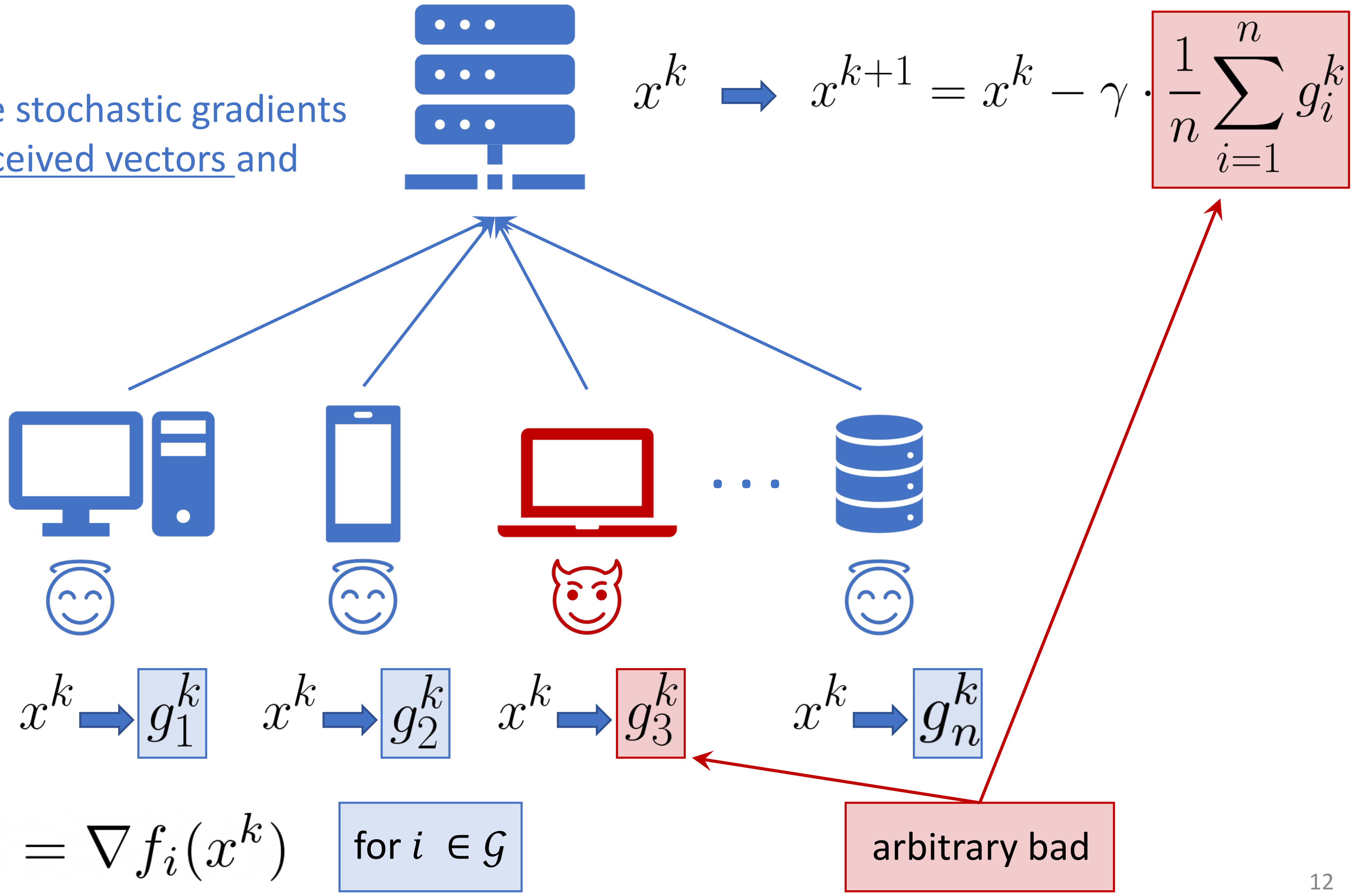
$$x^k \rightarrow x^{k+1} = x^k - \gamma \cdot \frac{1}{n}\sum_{i=1}^{n} g_i^k$$

$$\mathbb{E}_k\left[\boxed{g_i^k}\right] = \nabla f_i(x^k)$$

Parallel SGD Is Fragile

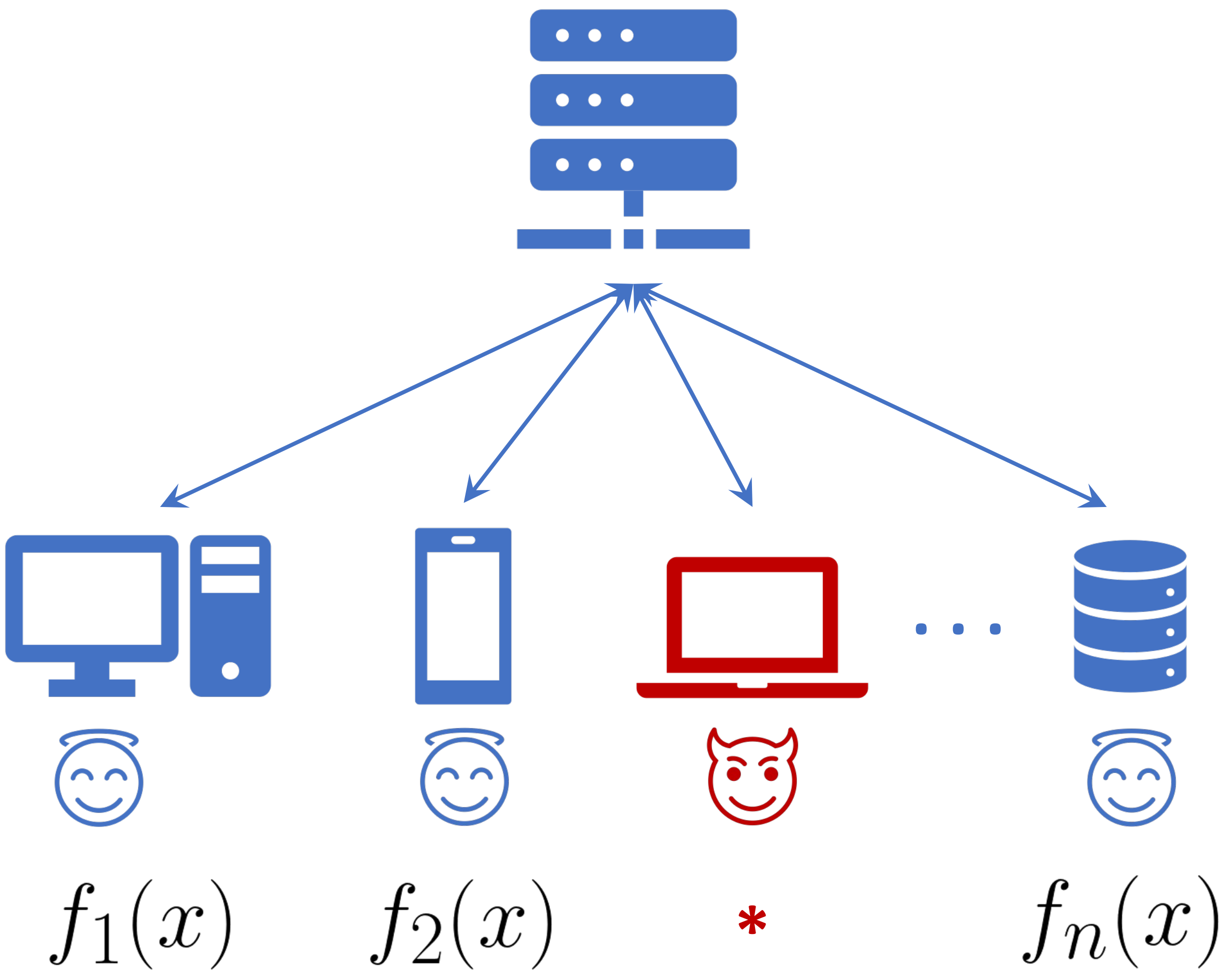
Iteration k:

1. Server broadcasts x^k
2. Good workers compute stochastic gradients
3. Server averages the received vectors and makes an SGD step



$\mathbb{E}_k[\boxed{g_i^k}] = \nabla f_i(x^k)$ for $i \in \mathcal{G}$

The Refined Problem Formulation



$$\min_{x \in \mathbb{R}^d} \left\{ f(x) := \frac{1}{G} \sum_{i \in \mathcal{G}} f_i(x) \right\}$$

Good workers form the majority:

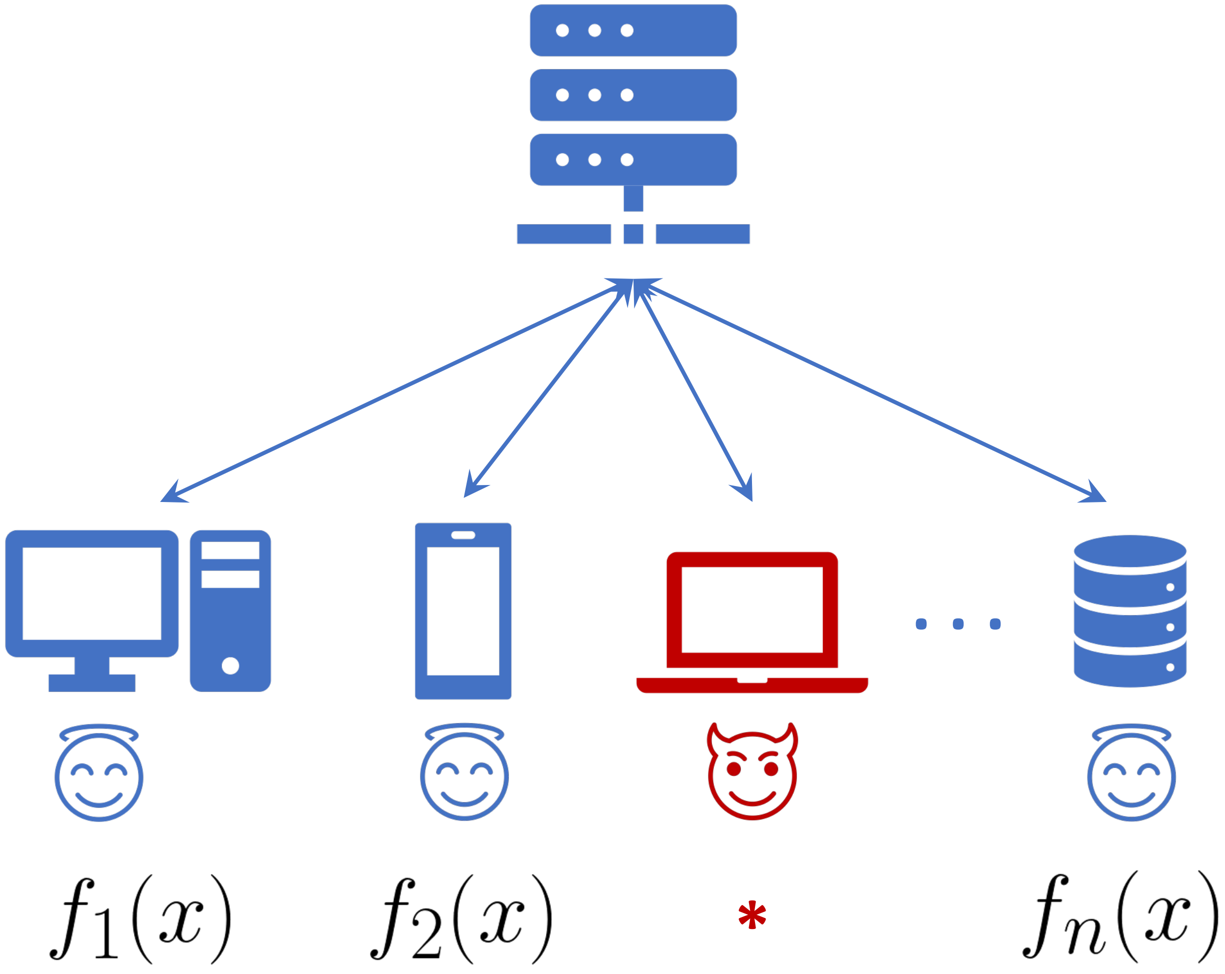
- $\mathcal{G}$ – good workers
- $\mathcal{B}$ – Byzantines (see the page "Byzantine fault" in Wikipedia)
- $\mathcal{G} \sqcup \mathcal{B} = [n], \quad |\mathcal{G}| = G, \quad |\mathcal{B}| = B$
- $B \leq \delta n, \quad \delta < 1/2$
- Byzantines are omniscient

On the heterogeneity:

- Loss functions on good peers cannot be arbitrary heterogeneous
- In this talk, we will assume that

$$\forall\, i \in \mathcal{G} \;\rightarrow\; f_i = f$$

The Refined Problem Formulation



Question: how to solve such problems?

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  $$\forall\, i \in \mathcal{G} \;\to\; f_i = f$$

Robust Aggregation

“Middle-Seekers” Aggregators

Natural idea: replace the averaging with more robust aggregation rule!

$$x^{k+1} = x^k - \gamma g^k \quad \Rightarrow \quad x^{k+1} = x^k - \gamma \widehat{g}^k$$

$$g^k = \frac{1}{n}\sum_{i=1}^{n} g_i^k \quad \Rightarrow \quad \widehat{g}^k = \mathtt{RAgg}\left(g_1^k, g_2^k, \ldots, g_n^k\right)$$

Question: how to choose aggregator?

“Middle-Seekers” Aggregators

- Geometric median (RFA):

Pillutla, K., Kakade, S. M., & Harchaoui, Z. (2019). Robust aggregation for federated learning. arXiv preprint arXiv:1912.13445.

$$\widehat{g}^k = \arg\min_{g \in \mathbb{R}^d} \sum_{i=1}^{n} \|g - g_i^k\|_2$$

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- Coordinate-wise median (CM):

Yin, D., Chen, Y., Kannan, R., & Bartlett, P. (2018, July). Byzantine-robust distributed learning: Towards optimal statistical rates. *In International Conference on Machine Learning* (pp. 5650-5659). PMLR.

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$$\widehat{g}^k = \arg\min_{g \in \mathbb{R}^d} \sum_{i=1}^{n} \|g - g_i^k\|_1$$

- Krum estimator:

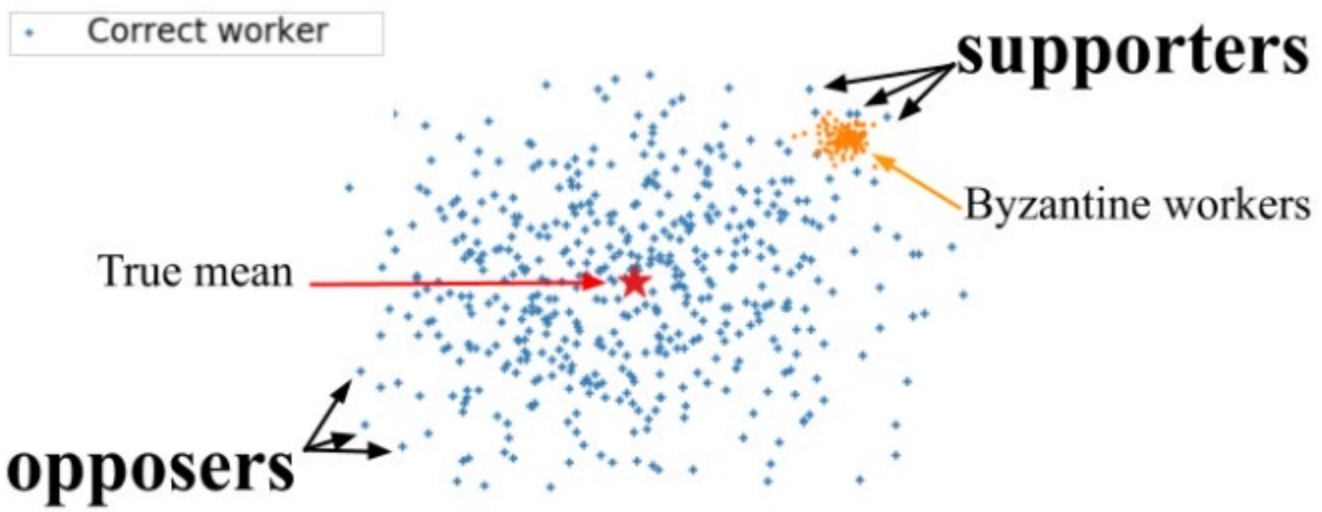
  Blanchard, P., El Mhamdi, E. M., Guerraoui, R., & Stainer, J. (2017, December). Machine learning with adversaries: Byzantine tolerant gradient descent. *In Proceedings of the 31st International Conference on Neural Information Processing Systems* (pp. 118-128).

$$\widehat{g}^k = \arg\min_{g \in \{g_1^k, \dots, g_n^k\}} \sum_{i \in \mathcal{N}_{n-B-2}(g)} \|g - g_i^k\|_2^2$$

indices of the closest $n - B -$ 2 workers to g

A Little Is Enough (ALIE) Attack

Baruch, G., Baruch, M., & Goldberg, Y. (2019). A little is enough: Circumventing defenses for distributed learning. *Advances in Neural Information Processing Systems*, 32.



Byzantines send the following vectors: $g_i^k = \boxed{\mu_{\mathcal{G}}} - z\boxed{\sigma_{\mathcal{G}}}$

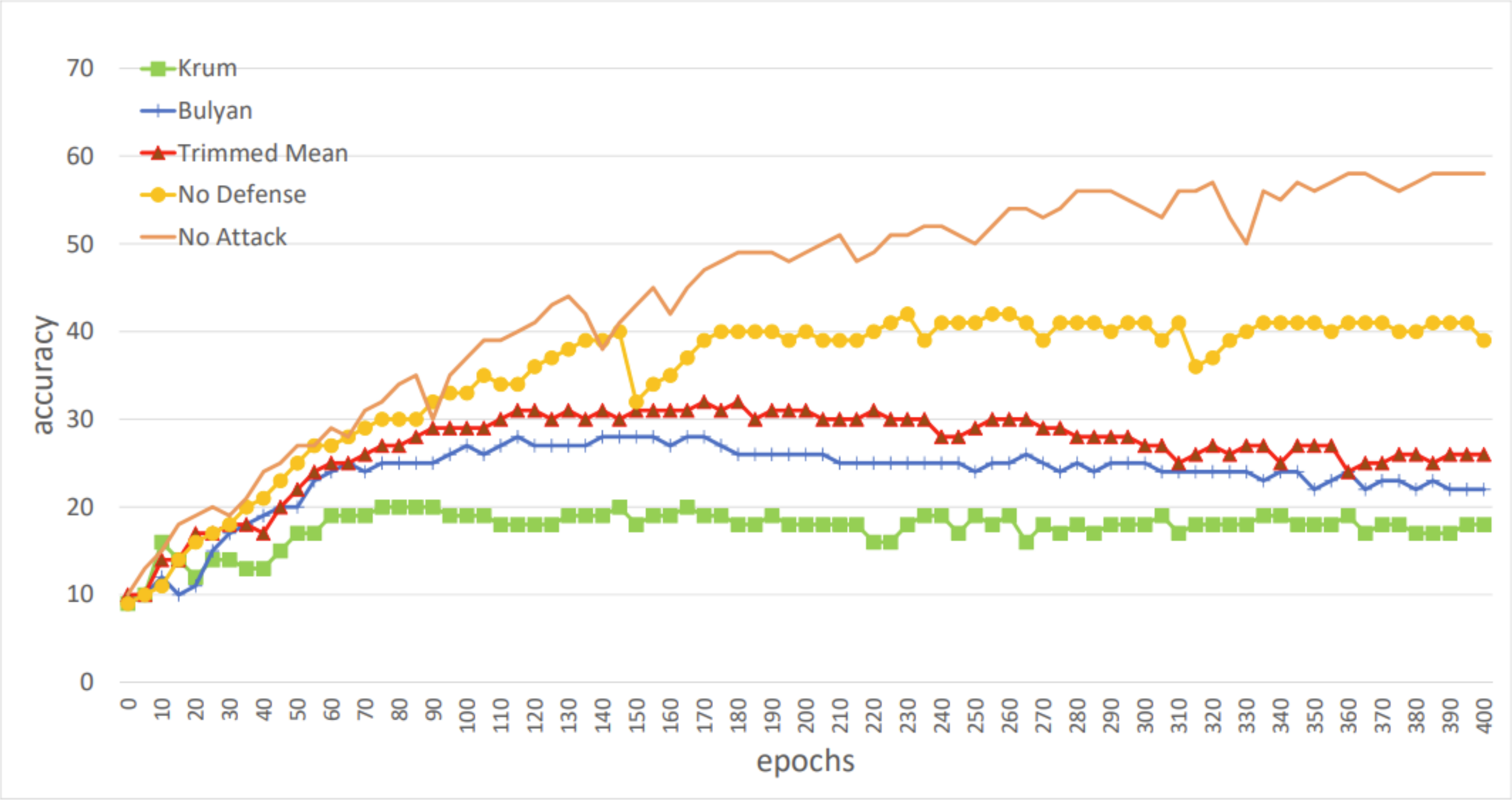
mean of the good workers

coordinate-wise standard deviation of good workers

- Byzantines choose z such that they are close to the "boundary of the cloud"
- Since Byzantines are closer to the mean, "middle-seekers" will treat opposers as outliers

The Result of ALIE Attack on the Training @ CIFAR10

Baruch, G., Baruch, M., & Goldberg, Y. (2019). A little is enough: Circumventing defenses for distributed learning. *Advances in Neural Information Processing Systems*, 32.



"No defense" strategy is more robust! Formal definition of robust aggregation is required!

Robust Aggregation Formalism

Karimireddy, S. P., He, L., & Jaggi, M. (2021, July). Learning from history for byzantine robust optimization. *In International Conference on Machine Learning* (pp. 5311-5319). PMLR.

Definition of (δ, c)–robust aggregator

Let $g_1 \dots, g_n$ be random variables such that there exist a good subset $\mathcal{G} \subseteq [n]$ of size $G \geq (1-\delta)n > {}^{n}/_{2}$ such that $\{g_i\}_{\{i \in \mathcal{G}\}}$ are independent and for all fixed pairs of good workers $i, j \in \mathcal{G}$ we have

$$\mathbb{E}\left[\|g_i - g_j\|^2\right] \leq \sigma^2.$$

Let $\bar{g} = \frac{1}{G}\sum_{i \in \mathcal{G}} g_i$. Then $\hat{g} = \mathrm{RAgg}(g_1, \dots, g_n)$ is called (δ, c)–robust aggregator if for some $c > 0$

$$\mathbb{E}\left[\|\widehat{g} - \overline{g}\|^2\right] \leq c\delta\sigma^2$$

- Medians and Krum estimators do not satisfy this definition
- **Question:** do such aggregators exist?

Bucketing Fixes "Middle-Seekers"

Karimireddy, S. P., He, L., & Jaggi, M. (2022). Byzantine-Robust Learning on Heterogeneous Datasets via Bucketing. *In International Conference on Learning Representations*.

Bucketing takes $\{g_1, \ldots, g_n\}$, positive integer s, and aggregator Aggr as an input and returns

$$\widehat{g} = \mathtt{Aggr}(y_1, \ldots, y_{\lceil n/s \rceil})$$

where $y_i = \frac{1}{s} \sum_{k=s(i-1)+1}^{\min\{si,n\}} x_{\pi(k)}$ and $\pi = (\pi(1), \ldots, \pi(n))$ is a random permutation of $[n]$

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For any $\delta \leq \delta_{\max}$ and $s = \lfloor {}^{\delta_{\max}}/_{\delta} \rfloor$

- Krum ∘ Bucketing is (δ, c)–robust aggregator with $c = \mathcal{O}(1)$ and $\delta_{\max} < {}^1/_4$
- RFA ∘ Bucketing is (δ, c)–robust aggregator with $c = \mathcal{O}(1)$ and $\delta_{\max} < {}^1/_2$
- CM ∘ Bucketing is (δ, c)–robust aggregator with $c = \mathcal{O}(d)$ and $\delta_{\max} < {}^1/_2$

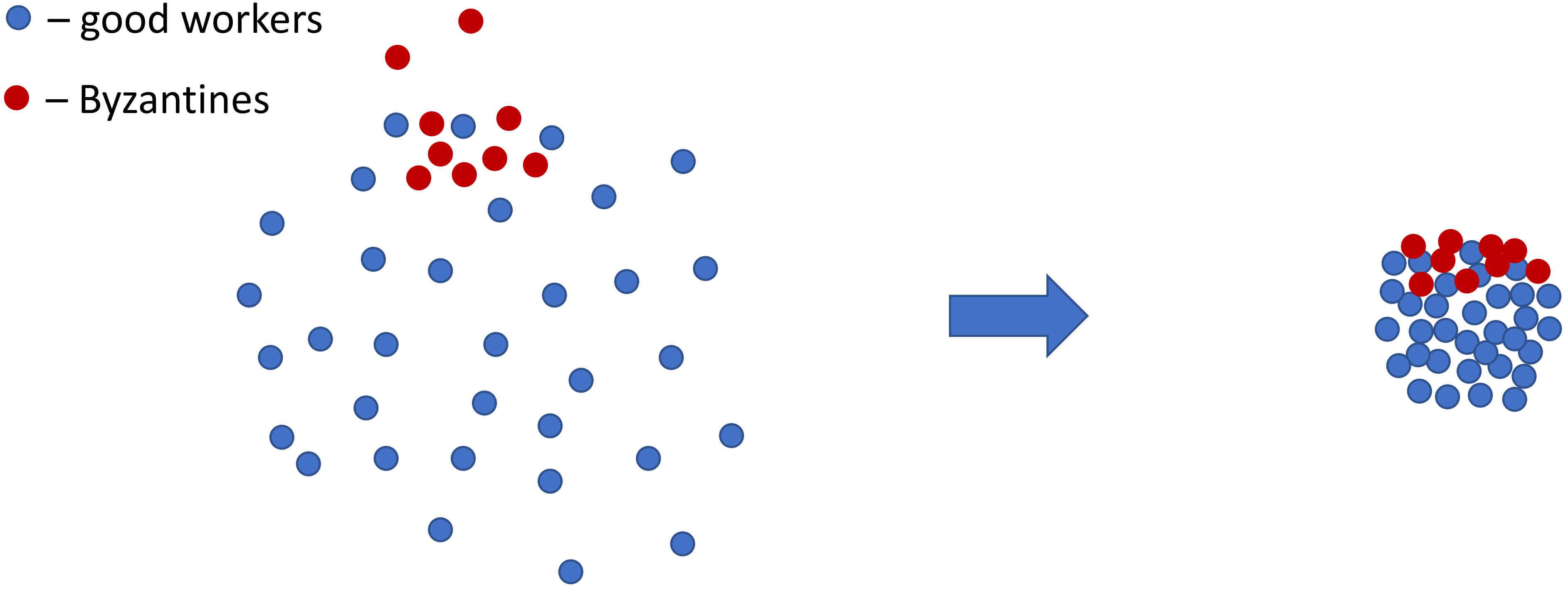
Moreover, these estimators are agnostic to σ^2!

Ingredient 1: Variance Reduction

Why Variance Reduction?

Wu, Z., Ling, Q., Chen, T., & Giannakis, G. B. (2020). Federated variance-reduced stochastic gradient descent with robustness to byzantine attacks. IEEE Transactions on Signal Processing, 68, 4583-4596.

Natural idea: if the variance of good vectors gets smaller, it becomes progressively harder for Byzantines to shift the result of the aggregation from the true mean



- **Large variance** allows Byzantines to hide in noise and still create large bias
- Hard to detect outliers

- **Small variance** does not allow Byzantines to create large bias easily
- Easy to detect outliers

Byrd-SAGA: Byzantine-Robust SAGA

Wu, Z., Ling, Q., Chen, T., & Giannakis, G. B. (2020). Federated variance-reduced stochastic gradient descent with robustness to byzantine attacks. IEEE Transactions on Signal Processing, 68, 4583-4596.

Finite-sum optimization:

$$\min_{x \in \mathbb{R}^d} \left\{ f(x) := \frac{1}{\boxed{m}} \sum_{j=1}^{\boxed{m}} \boxed{f_j(x)} \right\}$$

# of samples in the dataset

loss on *j*-th sample

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loss on j-th sample

Byrd-SAGA:

- Good workers compute SAGA-estimators
- Server uses geometric median aggregator

$$x^{k+1} = x^k - \gamma \widehat{g}^k$$

$$\widehat{g}^k = \mathtt{RFA}(g_1^k, \ldots, g_n^k)$$

$$g_i^k = \begin{cases} \nabla f_{j_{i_k}}(x^k) - \nabla f_{j_{i_k}}(\phi_{i,j_{i_k}}^k) + \frac{1}{m} \sum_{j=1}^{m} \nabla f_j(\phi_{i,j}^k), & \text{if } i \in \mathcal{G}, \\ *, & \text{if } i \in \mathcal{B} \end{cases}$$

$$\phi_{i,j}^{k+1} = \begin{cases} \phi_{i,j}^k, & \text{if } j \neq j_{i_k}, \\ x^k, & \text{if } j = j_{i_k} \end{cases} \quad \forall i \in \mathcal{G}$$

Complexity of Byrd-SAGA

Wu, Z., Ling, Q., Chen, T., & Giannakis, G. B. (2020). Federated variance-reduced stochastic gradient descent with robustness to byzantine attacks. IEEE Transactions on Signal Processing, 68, 4583-4596.

Assumptions:

- μ–strong convexity of f: $$f(y) \geq f(x) + \langle \nabla f(x), y - x \rangle + \frac{\mu}{2}\|y - x\|^2 \quad \forall x, y \in \mathbb{R}^d$$
- L–smoothness of $f_1, \ldots, f_m$: $$\|\nabla f_j(y) - \nabla f_j(x)\| \leq L\|y - x\| \quad \forall x, y \in \mathbb{R}^d, j \in [m]$$

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Theorem:

Let $\delta < {}^1/_2$ and the above assumptions hold. Then, there exists a choice of the stepsize γ such that the mini-batched version of Byrd-SAGA (with batchsize b) produces x^k satisfying $\mathbb{E}\left[\|x^k - x^*\|^2\right] \leq \varepsilon$ after

$$\mathcal{O}\left(\frac{m^2 L^2}{b^2(1-2\delta)\mu^2}\log\frac{1}{\varepsilon}\right) \quad \text{iterations}$$

Reflecting on the Complexities

- Complexity of Byrd-SAGA $(b = 1,\ \delta > 0)$: $$\mathcal{O}\left(\frac{m^2L^2}{(1-2\delta)\mu^2}\log\frac{1}{\varepsilon}\right)$$
- Complexity of Byrd-SAGA $(b = 1,\ \delta = 0)$: $$\mathcal{O}\left(\frac{m^2L^2}{\mu^2}\log\frac{1}{\varepsilon}\right)$$
- Complexity of SAGA $(b = 1,\ \delta = 0)$: $$\mathcal{O}\left(\left(m+\frac{L}{\mu}\right)\log\frac{1}{\varepsilon}\right)$$

Reflecting on the Complexities

- Complexity of Byrd-SAGA ($b = 1,\ \delta > 0$): $$\mathcal{O}\left(\frac{m^2L^2}{(1-2\delta)\mu^2}\log\frac{1}{\varepsilon}\right)$$
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- Complexity of SAGA ($b = 1,\ \delta = 0$): $$\mathcal{O}\left(\left(m + \frac{L}{\mu}\right)\log\frac{1}{\varepsilon}\right)$$

The reason for such a dramatic deterioration in the complexity of Byrd-SAGA in comparison to SAGA:

$$\mathbb{E}_k[\widehat{g}^k] \neq \nabla f(x^k)$$

Analysis of SAGA/SVRG-based methods is very sensitive to unbiasedness!

Biased VR: You Cannot “Break” What Is Already “Broken”!

SARAH/Geom-SARAH/PAGE (1 node case):

$$x^{k+1} = x^k - \gamma g^k$$

Nguyen, L. M., Liu, J., Scheinberg, K., & Takáč, M. (2017, July). SARAH: A novel method for machine learning problems using stochastic recursive gradient. In International Conference on Machine Learning (pp. 2613-2621). PMLR.

Horváth, S., Lei, L., Richtárik, P., & Jordan, M. I. (2022). Adaptivity of stochastic gradient methods for nonconvex optimization. SIAM Journal on Mathematics of Data Science, 4(2), 634-648.

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J_k– indices in the mini-batch, $|J_k| = b$

$$\boxed{\mathbb{E}_k[g^k] \neq \nabla f(x^k)}$$

Estimator is biased from the beginning!

Nguyen, L. M., Liu, J., Scheinberg, K., & Takáč, M. (2017, July). SARAH: A novel method for machine learning problems using stochastic recursive gradient. In International Conference on Machine Learning (pp. 2613-2621). PMLR.

Horváth, S., Lei, L., Richtárik, P., & Jordan, M. I. (2022). Adaptivity of stochastic gradient methods for nonconvex optimization. SIAM Journal on Mathematics of Data Science, 4(2), 634-648.

Li, Z., Bao, H., Zhang, X., & Richtárik, P. (2021, July). PAGE: A simple and optimal probabilistic gradient estimator for nonconvex optimization. In International Conference on Machine Learning (pp. 6286-6295). PMLR.

Byz-PAGE

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Geom-SARAH/PAGE–estimator

The method achieves theoretical SOTA rates but uses full participation of clients

E. Gorbunov, S. Horváth, P. Richtárik, G. Gidel. *Variance Reduction is an Antidote to Byzantines: Better Rates, Weaker Assumptions and Communication Compression as a Cherry on the Top* (ICLR 2023)

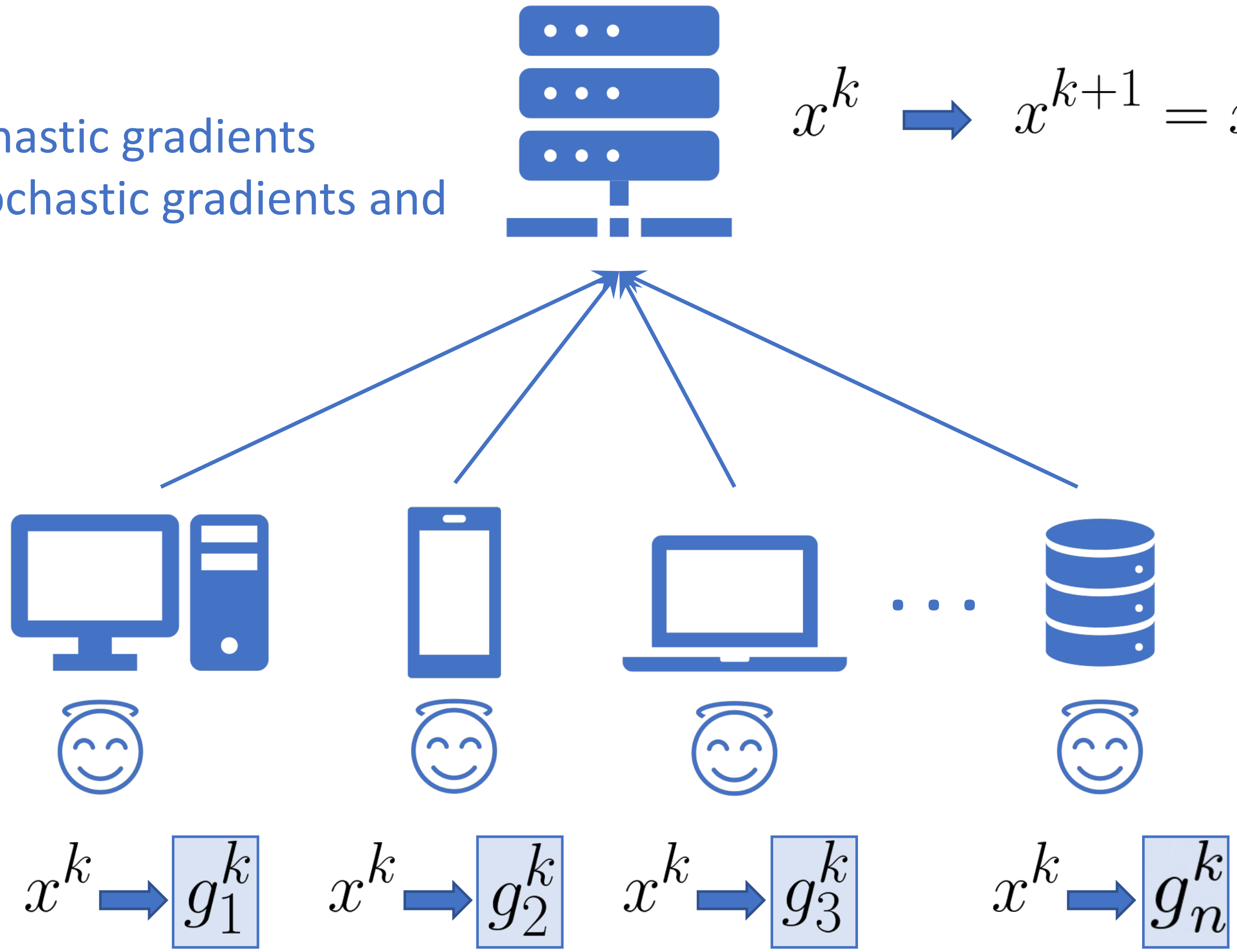
Partial Participation

Parallel SGD

Iteration k:

1. Server broadcasts x^k
2. Workers compute stochastic gradients
3. Server averages the stochastic gradients and makes an SGD step

$$x^k \Rightarrow x^{k+1} = x^k - \gamma \cdot \frac{1}{n}\sum_{i=1}^{n} g_i^k$$



$$\mathbb{E}_k\left[\boxed{g_i^k}\right] = \nabla f_i(x^k)$$

Parallel SGD with Partial Participation of Clients

Iteration k:

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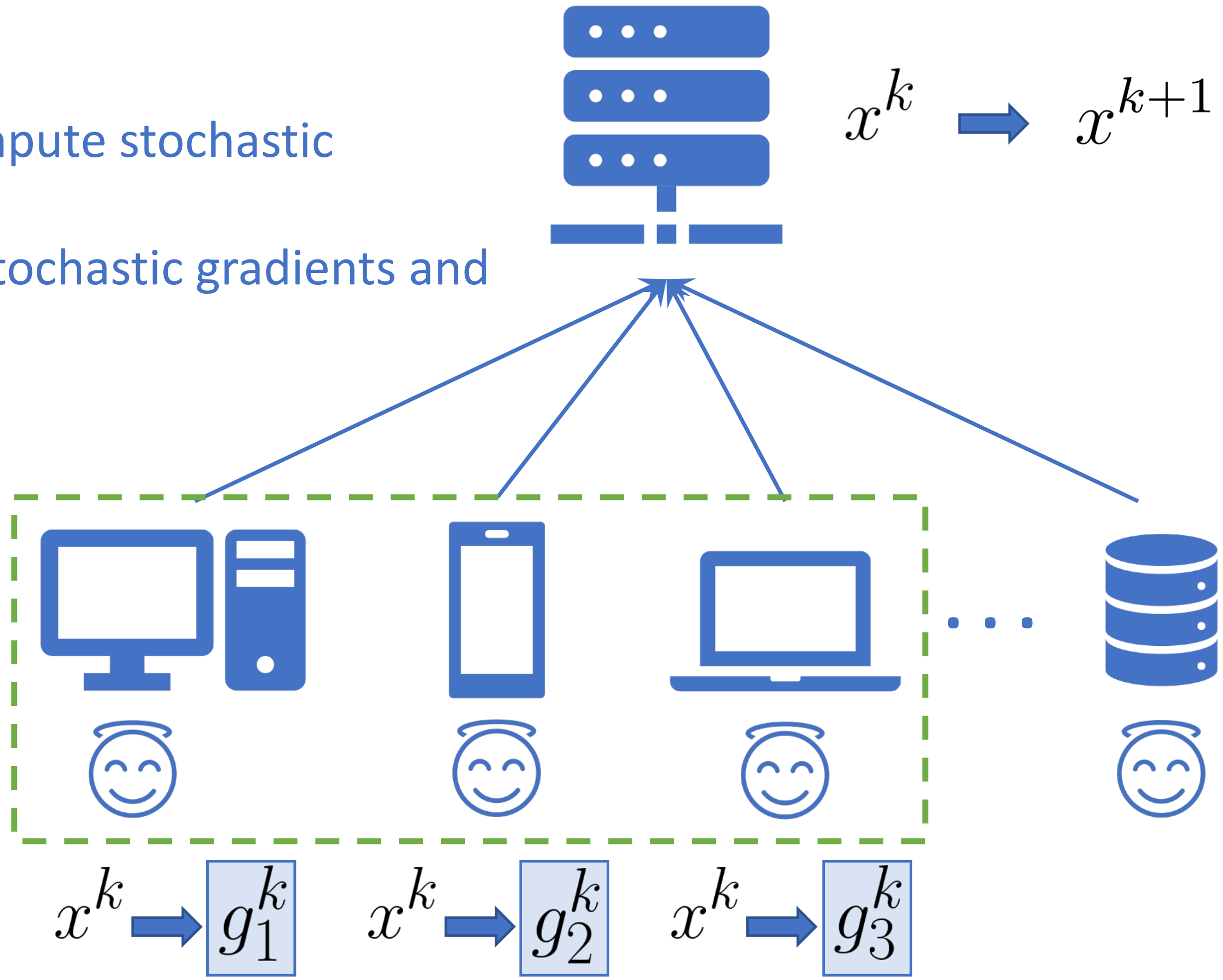


$$x^k \rightarrow x^{k+1} = x^k - \gamma \cdot \frac{1}{3}\sum_{i=1}^{3} g_i^k$$

$$\mathbb{E}_k[\boxed{g_i^k}] = \nabla f_i(x^k)$$

Why is it used?

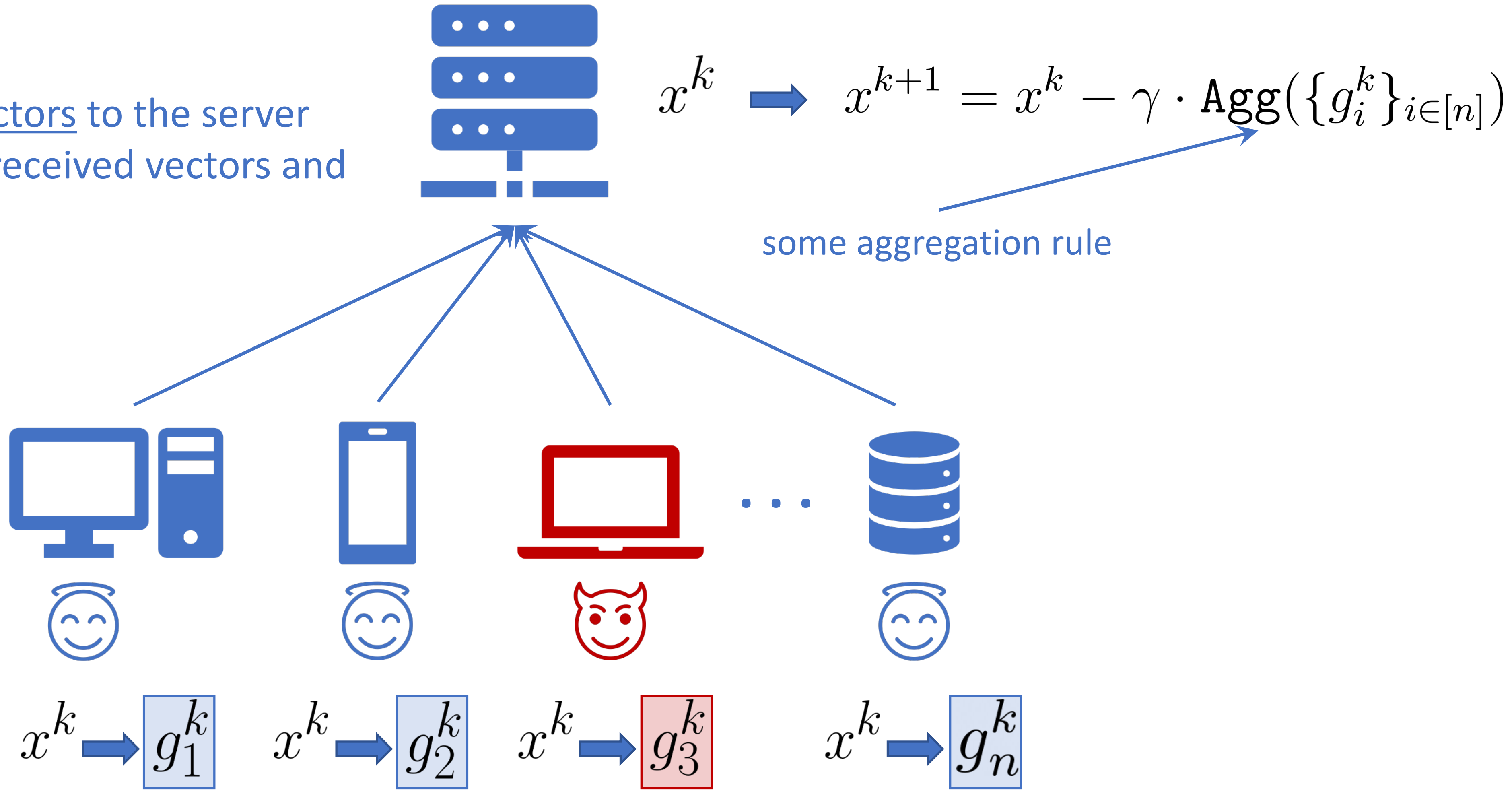
Clients sampling may speed up the training

Some clients may be unavailable at certain moments (poor connection, low battery, no free compute power)

Byzantine-Robust Method

Iteration k:

1. Server broadcasts x^k
2. Workers send <u>some vectors</u> to the server
3. Server <u>aggregates</u> the received vectors and makes an SGD step



Byzantine-Robust Method with Partial Participation

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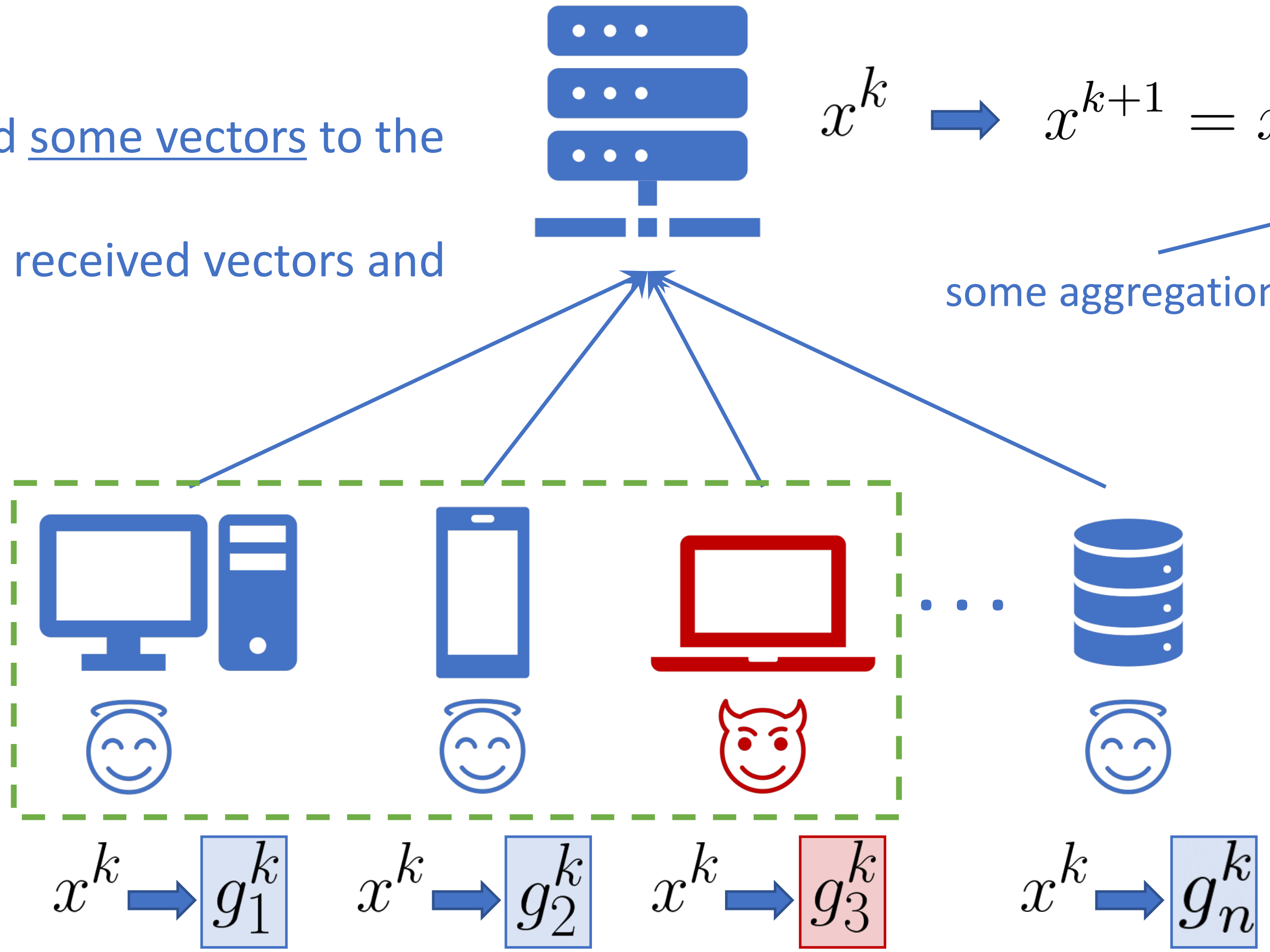

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some aggregation rule

sampled workers

If the majority of sampled workers are honest, the method works

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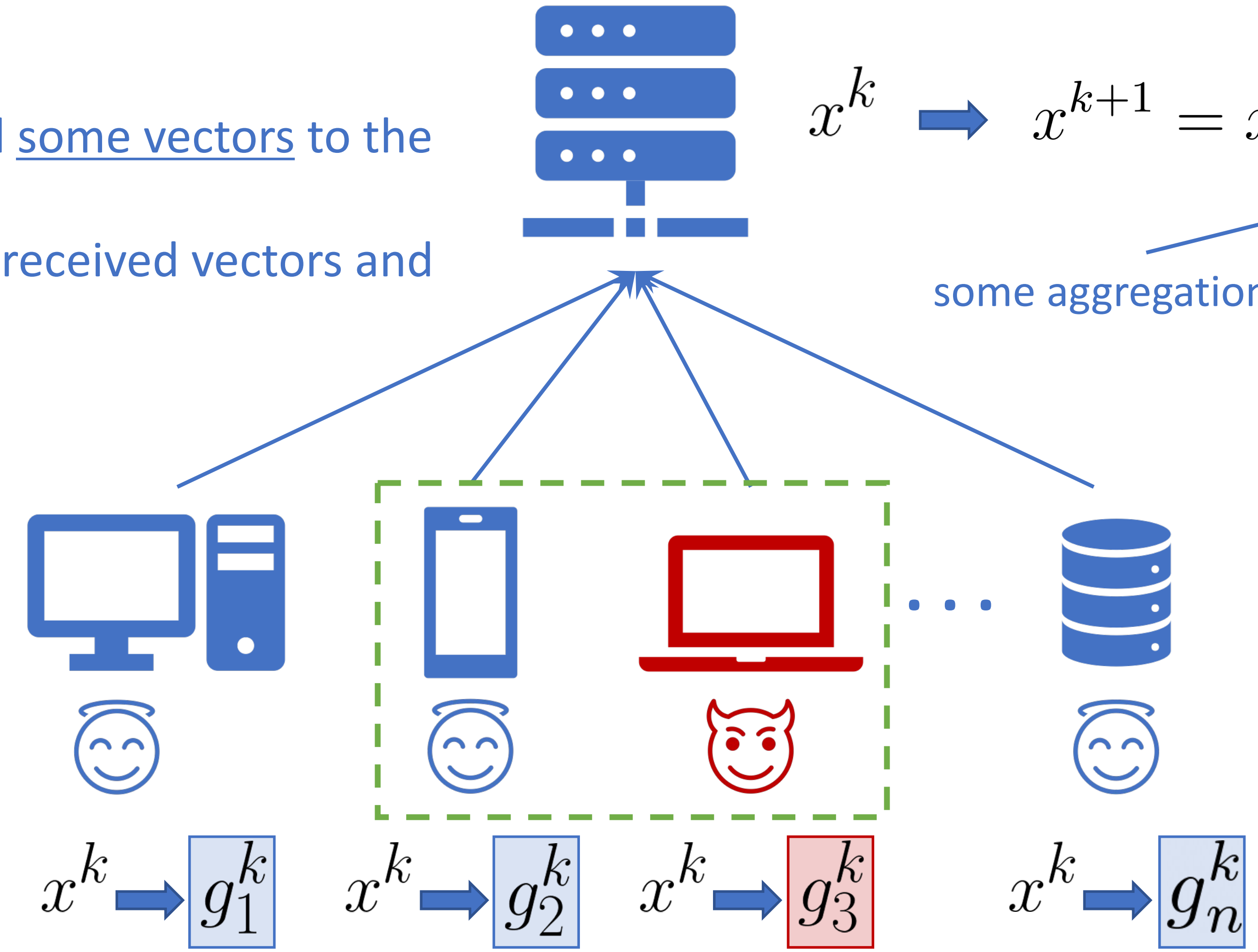

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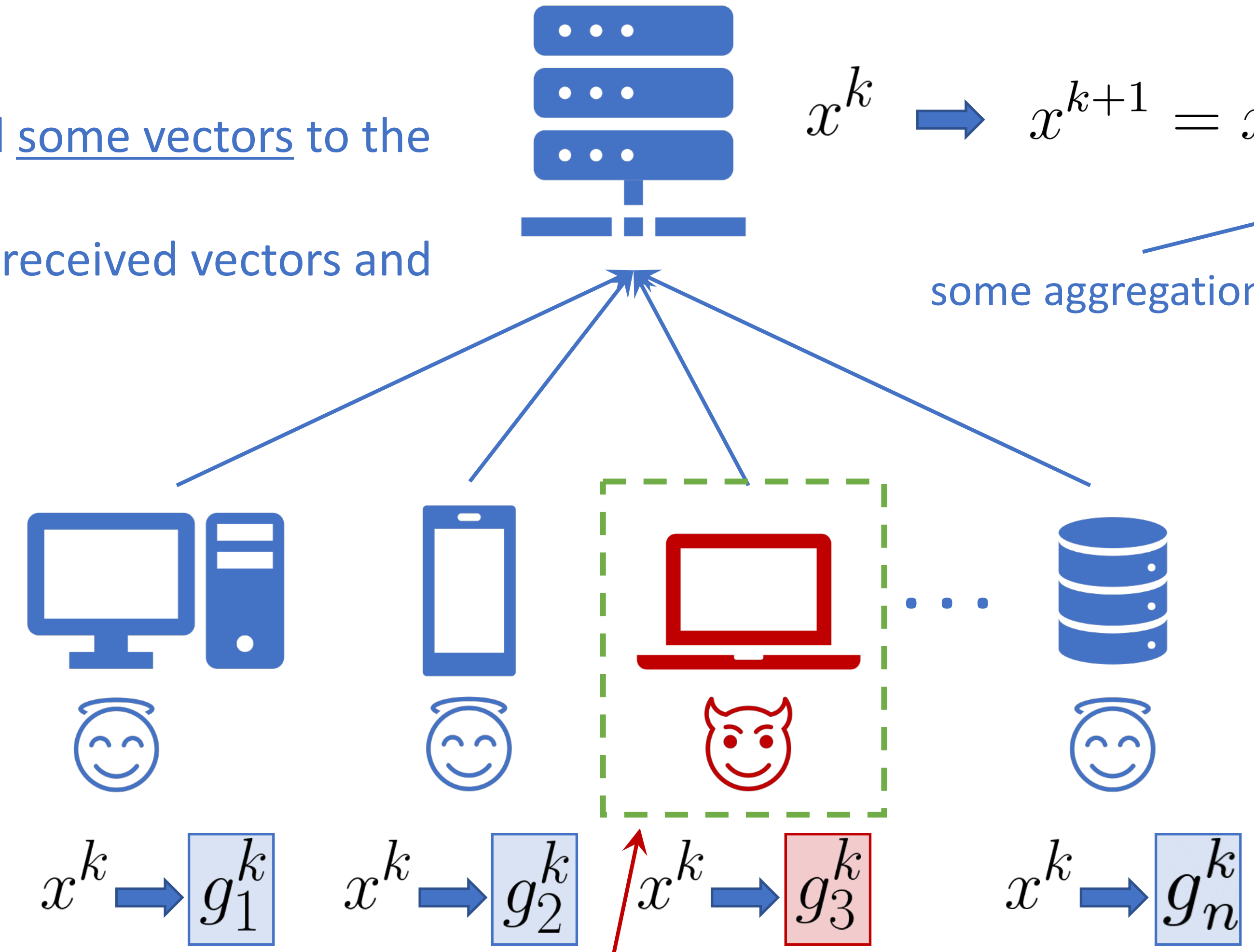

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The worst situation: all sampled workers are Byzantines

Ingredient 2: Clipping

Clipping Operator

💡 **Natural idea:** make all updates bounded via clipping

$$\mathrm{clip}(x, \lambda) = \begin{cases} \min\left\{1, \frac{\lambda}{\|x\|}\right\} x, & \text{if } x \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

Useful properties:

Boundeness $$\|\mathrm{clip}(x, \lambda)\| \leq \lambda$$

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Direction is preserved

New Method

New Method: Byz-PAGE-PP

💡 **Key idea:** clip gradient differences with $\boxed{\lambda_k \sim \|x^k - x^{k-1}\|}$

$$g_i^{k+1} = \begin{cases} \nabla f_i(x^{k+1}), & \text{with prob. } p \\ g^k + \boxed{\text{clip}\left(\frac{1}{b}\sum_{j\in J_k}(\nabla f_j(x^k) - \nabla f_j(x^{k-1})), \lambda_k\right)}, & \text{with prob. } 1-p \end{cases} \quad \forall i \in \mathcal{G}$$

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$$|S_k| = \begin{cases} \widehat{C}, & \text{with prob. } p, \\ C, & \text{with prob. } 1-p \end{cases}$$

$$\max\left\{1, \frac{\delta_{\text{real}} n}{\delta}\right\} \leq \widehat{C} \leq n$$

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Complexity of Byz-PAGE-PP (Simplified)

Assumptions:

- f is lower-bounded: $$f_* = \inf_{x \in \mathbb{R}^d} f(x) > -\infty$$
- L–smoothness of $f_1, \dots, f_m$: $$\|\nabla f_j(y) - \nabla f_j(x)\| \leq L\|y - x\| \quad \forall x, y \in \mathbb{R}^d, j \in [m]$$

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Let the above assumptions hold and ARAggr be (δ, c)–robust aggregator. Then, there exists a choice of the stepsize γ such that Byz-PAGE produces $\hat{x}^k$ satisfying $\mathbb{E}\left[\left\|\nabla f(\hat{x}^k)\right\|^2\right] \leq \varepsilon^2$ after

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iterations

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$$p_G = \mathrm{Prob}\{G_C^k \geq (1-\delta)C\}$$
$$\mathcal{P}_{\mathcal{G}_C^k} = \mathrm{Prob}\left\{i \in \mathcal{G}_C^k \mid G_C^k \geq (1-\delta)\, C\right\}$$

$F_{\mathcal{A}}$ - aggregation-dependent constant

Byz-PAGE vs Byz-PAGE-PP

Byz-PAGE-PP:

$$\mathcal{O}\left(\frac{\left(1+\sqrt{\frac{p_G G \mathcal{P}_{\mathcal{G}_C^k}}{pC}\left(\frac{1}{C}+\frac{c\delta}{p}\right)+\frac{(1-p_G)(1+F_{\mathcal{A}}^2)}{p^2}}\right)L\left(f(x^0)-f_*\right)}{\varepsilon^2}\right)$$

Byz-PAGE:

$$\mathcal{O}\left(\frac{\left(1+\sqrt{\frac{1}{p}\left(\frac{1}{n}+\frac{c\delta}{p}\right)}\right)L\left(f(x^0)-f_*\right)}{\varepsilon^2}\right)$$

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Matching results when all clients participate

Byz-PAGE vs Byz-PAGE-PP

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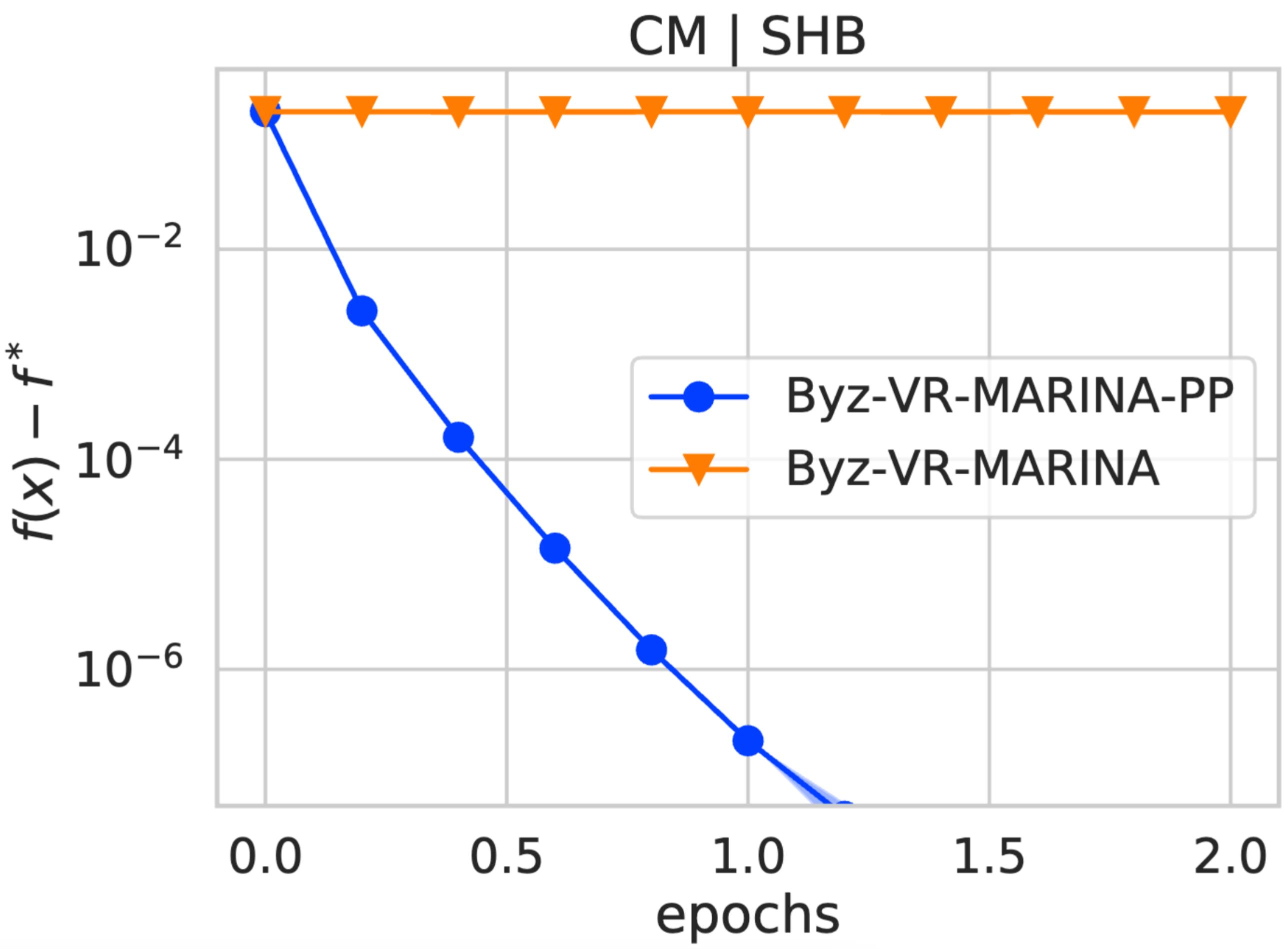
Matching results when all clients participate

When $p_G = 1$ (C is large enough) and $c\delta \geq p/C$, complexities are the same,
while Byz-PAGE-PP uses only $C \leq n$ workers at each step (on average) → provable benefits of PP!

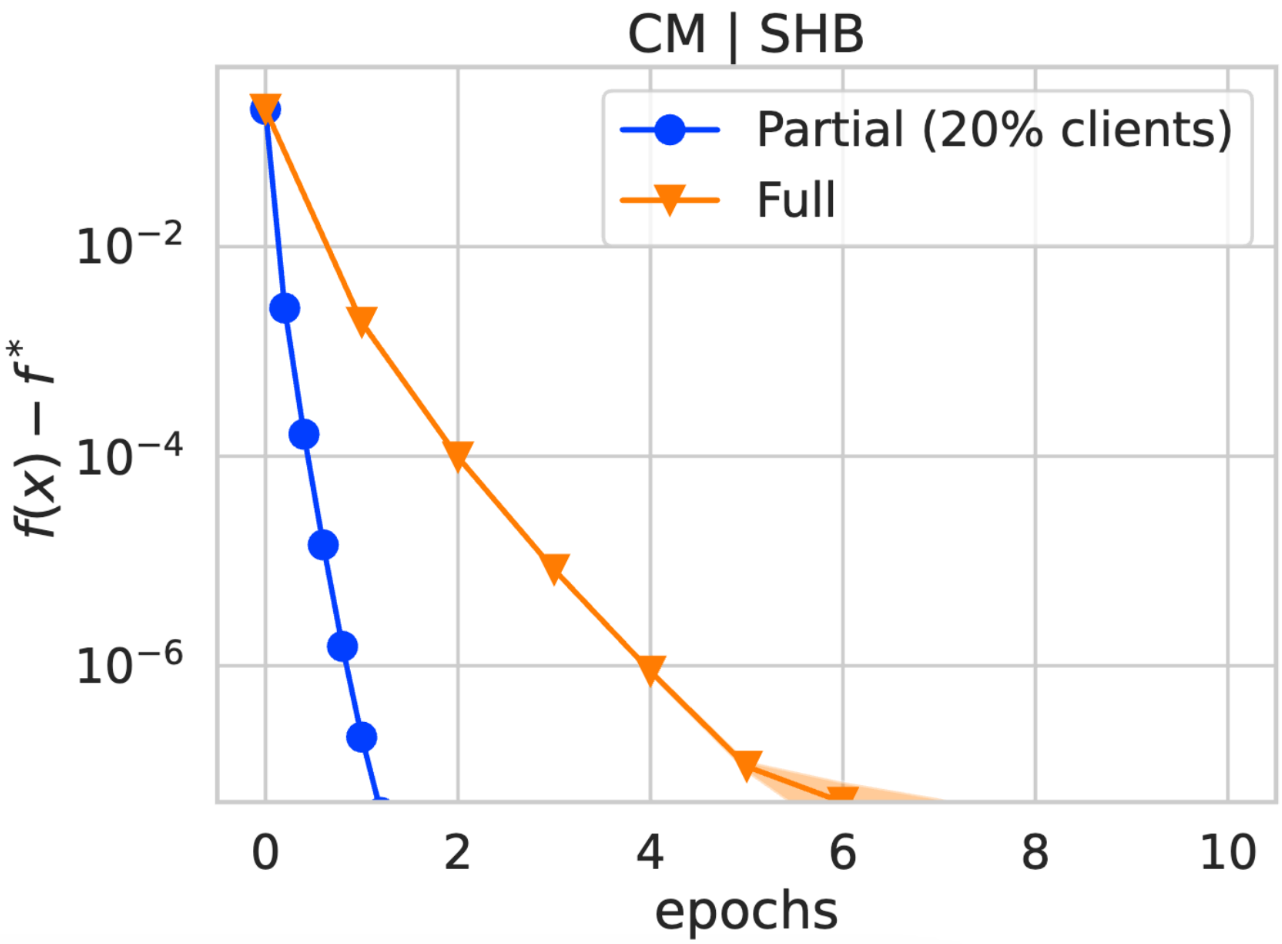
Numerical Results: Logistic Regression

- We tested the proposed method on the logistic regression tasks
- In this experiment, we have 15 good workers and 5 Byzantines
- Shift-back attack (SHB): when Byzantines form a majority they send $x^0 - x^k$
- Aggregation rule: coordinate-wise median (CM) with Bucketing
- Each round we sample 4 clients



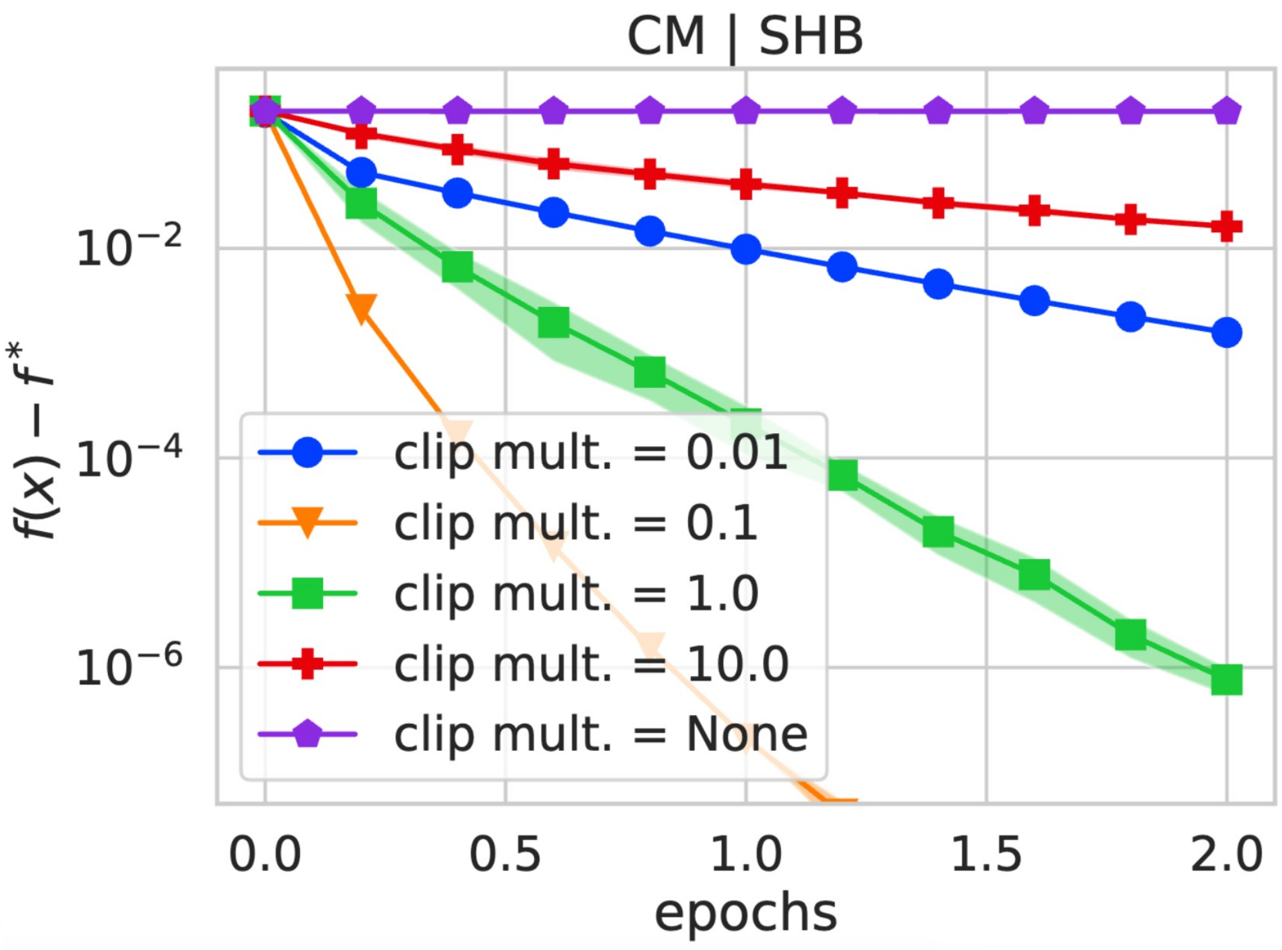
Numerical Results: Benefits of PP

- The method benefits from partial participation



Numerical Results: Sensivity to Clipping Level

- We also tested our method with different clipping multipliers λ: $\lambda_k = \lambda\|x^k - x^{k-1}\|$
- The method converges for different clipping values, though the speed depends on λ



Heuristic Extension

🤔 How to adjust any Byzantine-robust method to the case of Partial Participation?

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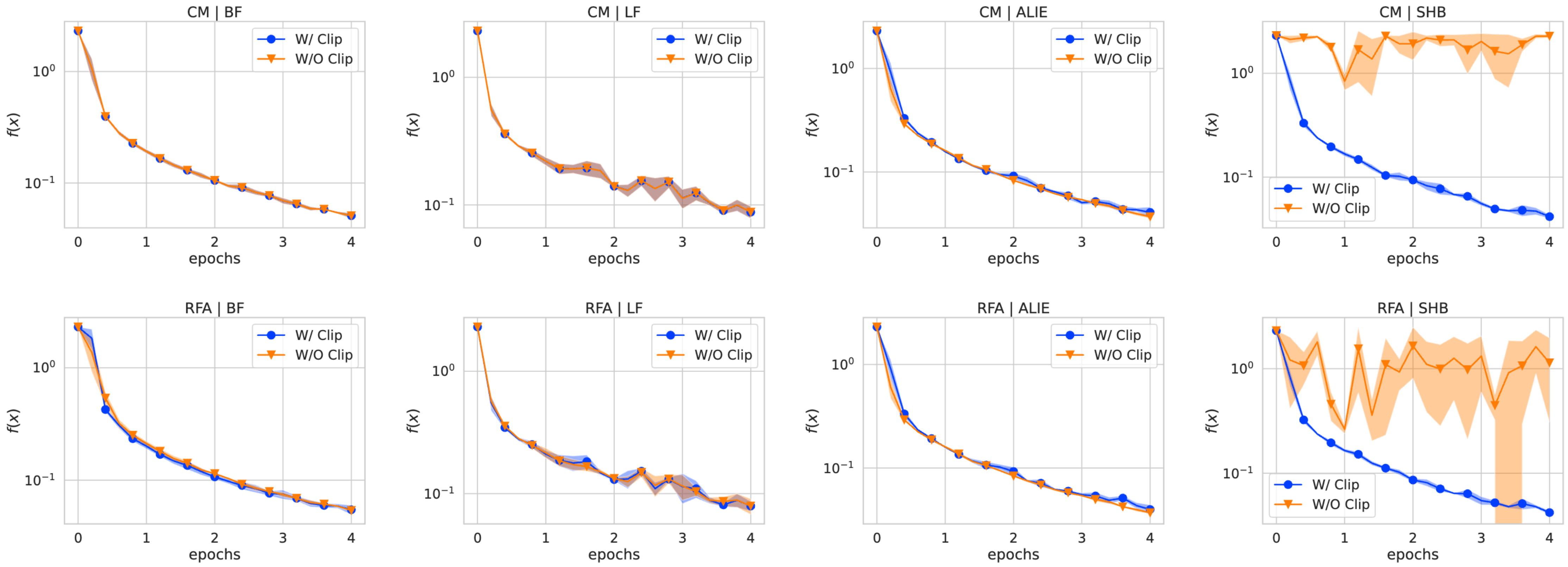
✓ We recommend to use $\boxed{\lambda_k = \lambda \|x^k - x^{k-1}\|}$ and tune λ in practice

Numerical Results: Neural Network Training

- We follow the setup from (Karimireddy et al., 2021) and train a certain NN on MNIST (LeCun and Cortes, 1998)
- In this experiment, we have 15 good workers and 5 Byzantines
- Attacks: A Little is Enough (ALIE) (Baruch et al., 2019), Bit Flipping (BF), Label Flipping (LF), Shift-Back (SHB)
- Aggregation rules: coordinate-wise median (CM), geometric median (RFA) with bucketing
- Each round we sample 4 clients
- Optimization method: Robust Momentum SGD (Karimireddy et al., 2021)

Karimireddy, S. P., He, L., Jaggi, M. *Learning from history for byzantine robust optimization* (ICML 2021)
LeCun, Y. and Cortes, C. *The MNIST database of handwritten digits* (http://yann.lecun.com/exdb/mnist/, 1998)
Baruch, G., Baruch, M., Goldberg, Y. *A Little is Enough: Circumventing defenses for distributed learning* (NeurIPS 2019)

Numerical Results: Neural Network Training



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- Clipping helps when Byzantine workers form majority (see SHB attack)

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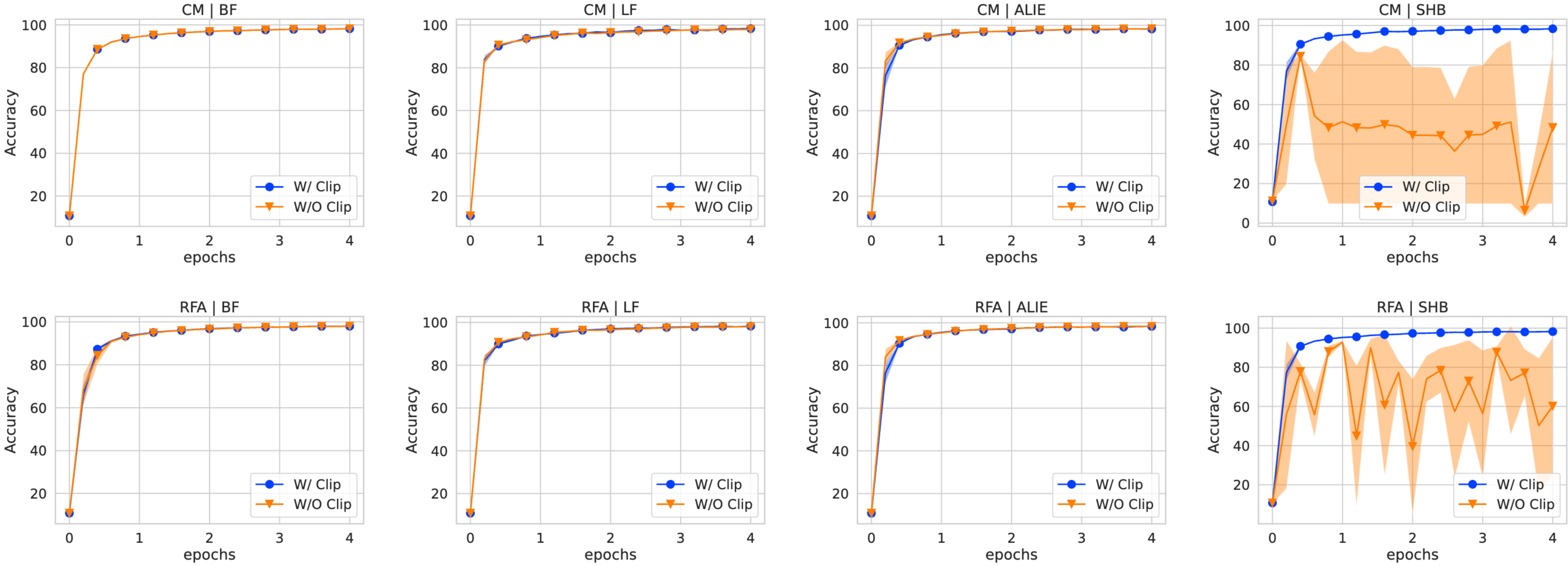

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Concluding Remarks

In the Paper We Also Have

- Analysis of the version with compression (Byz-VR-MARINA-PP)
- Analysis under bounded heterogeneity
- Non-uniform sampling of stochastic gradients
- Analysis taking into account data-similarity

Thank you!